

Aufgabe 1:

$$a) f(x) = \frac{\sin(2ax)}{2x+3}$$

$$f'(x) = \frac{2a \cos(2ax)(2x+3) - 2 \sin(2ax)}{(2x+3)^2}$$

$$b) f(x) = x \sin\left(\frac{1}{2}x^2 + 2\right)$$

$$f'(x) = \sin\left(\frac{1}{2}x^2 + 2\right) + x \cdot x \cdot \cos\left(\frac{1}{2}x^2 + 2\right)$$

$$c) f(x) = \frac{2x^2 + 2x - 1}{x^2 - 1}$$

$$f'(x) = \frac{(4x+2)(x^2-1) - 2x(2x^2+2x-1)}{(x^2-1)^2} = \frac{-2x^2 - 2x - 2}{(x^2-1)^2}$$

Aufgabe 2

$$a) f(x) = x^3 - e^{2x-2}$$

$$F(x) = \frac{1}{4}x^4 - \frac{1}{2}e^{2x-2} + C$$

$$F(1) = \frac{1}{4} - \frac{1}{2}e^0 + C = 1$$

$$-\frac{1}{4} + C = 1 \Rightarrow C = 1\frac{1}{4}$$

$$F(x) = \frac{1}{4}x^4 - \frac{1}{2}e^{2x-2} + \frac{5}{4}$$

$$b) \int_0^1 (e^{-2x} + 4)^2 dx = \int_0^1 (e^{-4x} + 8e^{-2x} + 16) dx$$

$$= \left[-\frac{1}{4}e^{-4x} - 4e^{-2x} + 16x \right]_0^1 = -\frac{1}{4}e^{-4} - 4e^{-2} + 16 - \left(-\frac{1}{4} - 4\right)$$

$$= -\frac{1}{4}e^{-4} - 4e^{-2} + 20\frac{1}{4}$$