

S 109 Nr. 1

$$\begin{aligned}
 \text{a) } A &= \int_{-2}^{-1} (x^2 - 1) dx + \int_{-1}^0 |x^2 - 1| dx = \int_{-2}^{-1} (x^2 - 1) dx + \int_{-1}^0 (-x^2 + 1) dx \\
 &= \left[\frac{x^3}{3} - x \right]_{-2}^{-1} + \left[-\frac{x^3}{3} + x \right]_{-1}^0 = -\frac{1}{3} + 1 - \left\{ -\frac{8}{3} + 2 \right\} + 0 - \left\{ \frac{1}{3} - 1 \right\} \\
 &= \frac{2}{3} + \frac{2}{3} + 0 + \frac{2}{3} = \frac{6}{3} = \underline{\underline{2}}
 \end{aligned}$$

$$\text{b) } f(x) = \frac{1}{x} - 2 \quad \text{Nullstellen: } \frac{1}{x} - 2 = 0 \Rightarrow \frac{1}{x} = 2 \Rightarrow x_N = 0,5$$

$$\begin{aligned}
 A &= \int_{0,5}^2 \left| \frac{1}{x} - 2 \right| dx = \int_{0,5}^2 \left(-\frac{1}{x} + 2 \right) dx = \left[-\ln|x| + 2x \right]_{0,5}^2 \\
 &= -\ln(2) + 2 \cdot 2 - \left\{ -\ln(0,5) + 1 \right\} = -\ln(2) + 4 + \ln(1) - \ln(2) - 1 \\
 &= 3 - 2\ln(2) \approx \underline{\underline{1,614}}
 \end{aligned}$$

$$\text{c) } f(x) = -x^2 + 1 = -3 \Rightarrow \text{Obere Integrationsgrenze}$$

$$x^2 = 4 \Rightarrow x_0 = +2$$

$$A = \int_0^2 (-x^2 + 1) - (-3) dx = \int_0^2 (-x^2 + 4) dx = \left[-\frac{x^3}{3} + 4x \right]_0^2$$

$$A = -\frac{8}{3} + 8 = \underline{\underline{\frac{16}{3} = 5,3}}$$

$$\text{d) } A = \int_0^1 (e^{x-1} - 1) dx + \int_1^2 (e^{x-1} - 1) dx = \int_0^1 (-e^{x-1} + 1) dx + \int_1^2 (e^{x-1} - 1) dx$$

$$A = \left[-e^{x-1} + x \right]_0^1 + \left[e^{x-1} - x \right]_1^2 = -e^0 + 1 - \left\{ -e^{-1} + 0 \right\} + e^1 - 2 - \left\{ e^0 - 1 \right\}$$

$$\underline{\underline{A = -1 + 1 + \frac{1}{e} + e - 2 - 0 = \frac{1}{e} + e - 2 \approx 1,086}}$$