S 110 Nr. 4

a) $g(x) > f(x)$ für $-1 \le x \le 1$

$$A = \int_{-1}^{1} (g(x) - f(x))\,dx = \int_{-1}^{1} (-x^2 + 4 - 0{,}5x)\,dx$$

$$A = \left[-\frac{x^3}{3} + 4x - \frac{1}{2} \cdot \frac{x^2}{2} \right]_{-1}^{1} = -\frac{1}{3} + 4 - \frac{1}{4} - \left\{ \frac{1}{3} - 4 - \frac{1}{4} \right\} =$$

$$= \frac{22}{3} \approx 7{,}333$$

---

b) $g(x) \ge f(x)$ für $0 \le x \le 1$

$$A = \int_{0}^{1} (x - x^3)\,dx = \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_{0}^{1} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

S 110 Nr. 5.

a) $f(x) = x^2$; $g(x) = -x^2 + 4x$

Schnitt der Graphen $\Rightarrow$ $x^2 = -x^2 + 4x \Rightarrow 0 = -2x^2 + 4x$

$\Rightarrow x(-2x + 4) = 0 \Rightarrow x_{s_1} = 0 \vee x_{s_2} = 2$

$g(x) > f(x)$ für $0 < x < 2$

$$\Rightarrow A = \int_{0}^{2} (-x^2 + 4x - x^2)\,dx = \int_{0}^{2} (-2x^2 + 4x)\,dx = \left[-2 \cdot \frac{x^3}{3} + 4 \cdot \frac{x^2}{2} \right]_{0}^{2}$$

$$A = -2 \cdot \frac{8}{3} + 4 \cdot 2 = -\frac{16}{3} + 8 = \frac{8}{3}$$

---

b) Schnitt der Graphen $\Rightarrow$ $-\frac{1}{x^2} = 2{,}5x - 5{,}25 \mid \cdot x^2 \Rightarrow -1 = 2{,}5x^3 - 5{,}25x^2$

$x_{s_1} = 0{,}5 \vee x_{s_2} = 2$ mit GTR ; $f(x) > g(x)$ für $0{,}5 < x < 2$

$$\Rightarrow A = \int_{0{,}5}^{2} \left(-\frac{1}{x^2} - (2{,}5x - 5{,}25) \right) dx = \int_{0{,}5}^{2} \left(-\frac{1}{x^2} - 2{,}5x + 5{,}25 \right) dx =$$

$$A = \left[\frac{1}{x} - 2{,}5 \cdot \frac{x^2}{2} + 5{,}25x \right]_{0{,}5}^{2} = \frac{1}{2} - 2{,}5 \cdot 2 + 5{,}25 \cdot 2 - \left\{ 2 - \frac{2{,}5}{8} + \frac{5{,}25}{2} \right\}$$

$$= \frac{27}{16} \approx 1{,}688$$