S 215 Nr. 11

d)
$$\begin{array}{rcl|l} 3x_1 + 3x_2 - 5x_3 &=& 3r & \cdot 1 \\ x_1 + 6x_2 - 10x_3 &=& r & \cdot(-3) \\ 15x_2 + 25x_3 &=& 0 & \end{array}$$

$$\begin{array}{rcl|l} x_1 + 6x_2 - 10x_3 &=& r & \\ -15x_2 + 25x_3 &=& 0 & \cdot 1 \\ 15x_2 + 25x_3 &=& 0 & \cdot 1 \end{array}$$

$x_1 + 6x_2 - 10x_3 = r \Rightarrow x_1 + 0 + 0 = r \Rightarrow \underline{x_1 = r}$

$-15x_2 + 25x_3 = 0 \Rightarrow \underline{x_2 = 0}$

$50x_3 = 0 \Rightarrow \underline{x_3 = 0}$

$\mathbb{L} = \{(r; 0; 0)\}$

e)
$$\begin{array}{rcl|l|l} 3x_1 - 2x_2 + x_3 &=& 2r & \cdot 5 & \cdot 1 \\ 5x_1 - 4x_2 - x_3 &=& 2 & \cdot(-3) & \\ x_1 + 3x_2 - 2x_3 &=& 2r+6 & & \cdot(-3) \end{array}$$

$$\begin{array}{rcl|l} 3x_1 - 2x_2 + x_3 &=& 2r & \\ 2x_2 + 8x_3 &=& 10r - 6 & \cdot 11 \\ -11x_2 + 7x_3 &=& -4r - 18 & \cdot 2 \end{array}$$

$3x_1 - 2x_2 + x_3 = 2r$

$2x_2 + 8x_3 = 10r - 6$

$102x_3 = 102r - 102$

$\underline{x_3 = (102r - 102) \cdot \frac{1}{102} = r - 1}$

$2x_2 + 8(r-1) = 10r - 6 \Rightarrow 2x_2 = 2r + 2 \Rightarrow \underline{x_2 = r + 1}$

$3x_1 - 2(r+1) + r - 1 = 2r \Rightarrow 3x_1 = 2r + 2r - r + 2 + 1 = 3r + 3$

$\underline{x_1 = r + 1}$

$\mathbb{L} = \{(r+1; r+1; r-1)\}$