S. 307 Nr. 11

$g: \vec{x} = \begin{pmatrix} -5 \\ -2 \\ 6 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ $\quad h_a: \vec{x} = \begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix} + r \cdot \begin{pmatrix} -2 \\ 1 \\ a \end{pmatrix}$

a) wähle 2 beliebige a

$a_1 = 0 \quad a_2 = 1$

$E: \vec{x} = \begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix} + a \cdot \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + b \cdot \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$

b) $P(0|-3|0)$ $\quad R(-5+2t|-2+t|6-2t)$

$$\vec{PR} = \begin{pmatrix} -5+2t \\ -2+t+3 \\ 6-2t \end{pmatrix} = \begin{pmatrix} 2t-5 \\ t+1 \\ -2t+6 \end{pmatrix}$$

$$\begin{pmatrix} 2t-5 \\ t+1 \\ -2t+6 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \stackrel{!}{=} 0$$

$$2 \cdot (2t-5) + (t+1) - 2 \cdot (-2t+6) = 0$$

$$4t - 10 + t + 1 + 4t - 12 = 0$$

$$9t - 21 = 0 \quad |+21$$

$$9t = 21 \quad |:9$$

$$t = \frac{21}{9} = \frac{7}{3}$$

$$\vec{PR} = \begin{pmatrix} \frac{14}{3} - 5 \\ -\frac{9}{3} + 1 \\ -\frac{14}{3} + 6 \end{pmatrix} = \begin{pmatrix} \frac{14}{3} - \frac{15}{3} \\ -\frac{9}{3} + \frac{3}{3} \\ -\frac{14}{3} + \frac{18}{3} \end{pmatrix} = \begin{pmatrix} -\frac{1}{3} \\ -\frac{10}{3} \\ \frac{4}{3} \end{pmatrix}$$

$$|\vec{PR}| = \sqrt{\left(-\frac{1}{3}\right)^2 + \left(\frac{10}{3}\right)^2 + \left(\frac{4}{3}\right)^2} = \sqrt{13}$$

$$= \sqrt{\frac{1}{9} + \frac{100}{9} + \frac{16}{9}} = \sqrt{\frac{117}{9}}$$

c)

$$E^*: \vec{x} = \begin{pmatrix} -5 \\ -2 \\ 6 \end{pmatrix} + r \cdot \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} + s \cdot \begin{pmatrix} -2 \\ 1 \\ a \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \times \begin{pmatrix} -2 \\ 1 \\ a \end{pmatrix} \qquad \begin{matrix} x_1: \\ x_2: \\ x_3: \end{matrix} \begin{pmatrix} a+2 \\ 4-2a \\ 2+2 \end{pmatrix} \qquad \vec{n} = \begin{pmatrix} a+2 \\ -2a+4 \\ 4 \end{pmatrix}$$

$$E^*: (a+2)x_1 + (-2a+4)x_2 + 4x_3 = -5(a+2) - 2\cdot(-2a+4) + 24$$

$$(a+2)x_1 + (-2a+4)x_2 + 4x_3 = \underline{-5a} \; \underline{-10} \; \underline{+4a} \; \underline{-8} \; \underline{+24}$$

$$(a+2)x_1 + (-2a+4)x_2 + 4x_3 = -a + 6$$

P(0|-3|0)

HNF

$$\left| \frac{(a+2)\cdot 0 + (-2a+4)\cdot(-3) + 0\cdot 4 + a - 6}{\sqrt{(a+2)^2 + (-2a+4)^2 + 4^2}} \right| = d$$

$$\left| \frac{6a - 12 + a - 6}{\sqrt{(a+2)^2 + (4-2a)^2 + 16}} \right| = d$$

$$\left| \frac{7a - 18}{\sqrt{\underline{a^2} + \underline{4a} \; \underline{+4} + \underline{16} \; \underline{-16a} + \underline{4a^2} \; \underline{+16}}} \right| = d = \left| \frac{7a - 18}{\sqrt{5a^2 - 12a + 36}} \right|$$

d) wenn $d = 0$ $\Rightarrow$ Nenner wird nicht 0

$\Rightarrow$ $7a - 18 \overset{!}{=} 0 \quad | + 18$

$7a = 18 \quad | : 7$

$a = \frac{18}{7}$

$h: \vec{x} = \begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix} + s \cdot \begin{pmatrix} -2 \\ 1 \\ 18/7 \end{pmatrix}$

$g: \vec{x} = \begin{pmatrix} -5 \\ -2 \\ 6 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$

$-2s = -5 + 2t$

$-3 + s = -2 + t$

$\frac{18}{7}s = 6 - 2t$

$-2s - 2t = -5$ GTR $s = \frac{7}{4}$ $t = \frac{3}{4}$ ing

$s - t = 1$

$\frac{18}{7}s + 2t = 6$

$\vec{x} = \begin{pmatrix} -5 \\ -2 \\ 6 \end{pmatrix} + \frac{3}{4} \cdot \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} -5 + 1,5 \\ -2 + 0,75 \\ 6 - 1,5 \end{pmatrix} = \begin{pmatrix} -3,5 \\ -1,25 \\ 4,5 \end{pmatrix}$

$S(-3,5 \mid -1,25 \mid 4,5)$

e) $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 1 \\ a \end{pmatrix} \stackrel{!}{=} 0$

$-4 + 1 - 2a = 0 \quad | + 2a$

$2a = -3 \quad | : 2$

$a = -\frac{3}{2}$

$h: \vec{x} = \begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix} + s \cdot \begin{pmatrix} -2 \\ 1 \\ -\frac{3}{2} \end{pmatrix}$

$$E^*: \vec{x} = \begin{pmatrix} -5 \\ -2 \\ 6 \end{pmatrix} + a \cdot \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} + b \cdot \begin{pmatrix} -2 \\ 1 \\ -\frac{3}{2} \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \times \begin{pmatrix} -2 \\ 1 \\ -\frac{3}{2} \end{pmatrix} \qquad \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} \begin{pmatrix} -\frac{3}{2} & +2 \\ -4 & +3 \\ 2 & +2 \end{pmatrix} = \begin{pmatrix} 0{,}5 \\ 7 \\ 4 \end{pmatrix} \qquad \begin{pmatrix} 0{,}5 \\ 7 \\ 4 \end{pmatrix} \cdot 2 = \begin{pmatrix} 1 \\ 14 \\ 8 \end{pmatrix}$$

$$E^*: \quad x_1 + 14\,x_2 + 8\,x_3 = 15$$

$$d(P;E) = \left| \frac{0 + 14 \cdot (-3) + 8 \cdot 0 - 15}{\sqrt{1^2 + 14^2 + 8^2}} \right| = \left| \frac{-57}{\sqrt{261}} \right| \approx 3{,}53 \text{ LE}$$