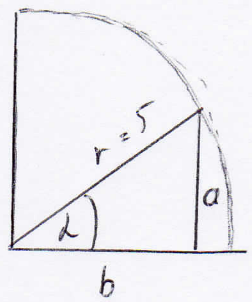
S 41 Nr. 9



$A_\Delta = \frac{1}{2} a \cdot b$

Nebenbedingung

$a^2 + b^2 = r^2 = 5^2$

$a^2 = 5^2 - b^2$

$a = \sqrt{5^2 - b^2}$

$A_\Delta(b) = \frac{1}{2} b \cdot \sqrt{5^2 - b^2}$

$A'(b) = \frac{1}{2} \cdot \sqrt{25 - b^2} + \frac{1}{2} b \cdot \frac{1}{2} (25 - b^2)^{-\frac{1}{2}} \cdot (-2b)$

$A'(b) = \frac{1}{2} \cdot \sqrt{25 - b^2} + \frac{-b^2}{2\sqrt{25 - b^2}} = \frac{1(25 - b^2) - b^2}{2 \cdot \sqrt{25 - b^2}}$

$A'(b) = \frac{25 - 2b^2}{\sqrt{25 - b^2}} = 0 \Rightarrow 2b^2 = 25$

$b^2 = 12,5$

$b = \sqrt{12,5} \Rightarrow a = \sqrt{25 - 12,5}$

$a = \sqrt{12,5}$

$\Rightarrow a = b = \sqrt{12,5} \Rightarrow \tan(\alpha) = \frac{a}{b} = \frac{a}{a} = 1 \Rightarrow \underline{\underline{\alpha = 45^\circ}}$

Gleiche Aufgabe Nebenbedingung aber anders gewählt!

$a = r \cdot \sin(\alpha)$; $b = r \cdot \cos(\alpha)$

$A(\alpha) = \frac{1}{2} a \cdot b = \frac{1}{2} r \cdot \sin(\alpha) \cdot r(\cos(\alpha)) = \frac{1}{2} r^2 \cdot \sin(\alpha) \cdot \cos(\alpha)$

Extrema $\Rightarrow$ $\cancel{A'(\alpha) = \frac{r^2}{2} \cdot \cos(\alpha) \cdot (-\sin(\alpha)) = -\frac{r^2}{2} \cdot \cos(\alpha) \cdot (\sin(\alpha))}$

$A'(\alpha) = \frac{1}{2} r^2 \left[\cos(\alpha) \cdot \cos(\alpha) + \sin(\alpha) \cdot (-\sin\alpha) \right] = \frac{1}{2} r^2 \cdot \left[(\cos(\alpha))^2 - (\sin(\alpha))^2 \right]$

$\Rightarrow \underline{\alpha = 45^\circ}$

Mit GTR kein Problem das Maximum von $\frac{1}{2} r^2 \cdot \sin(\alpha) \cdot \cos(\alpha) = A(\alpha)$ zu suchen.