

568 Nr. 9

a)  $f(x) = e^x + e^{-x} \Rightarrow f'(x) = e^x + e^{-x} \cdot (-1) = e^x - e^{-x}$

$f'(x) = e^x - \frac{1}{e^x} \stackrel{!}{=} \frac{e^x \cdot e^x - 1}{e^x} = \frac{e^{2x} - 1}{\underbrace{e^x}_{\neq 0}} = 0$  notw. Bed.

$\Rightarrow e^{2x} - 1 = 0 \Rightarrow e^{2x} = 1 \Rightarrow x_E = 0$

$f''(x) = e^x + e^{-x}$  ;  $f''(0) = e^0 + \frac{1}{e^0} = 1 + 1 = 2 > 0 \Rightarrow$  Min  
hinr. Bed.

T(0|2)

b)  $f(x) = -x + e^x$  ;  $f'(x) = -1 + e^x$  ;  $f''(x) = e^x$

notw. Bed. Extrema  $f'(x) = 0 = -1 + e^x \Rightarrow e^x = 1 \Rightarrow x_E = 0$

hinr. Bed  $f''(0) = e^0 = 1 > 0 \Rightarrow$  T(0|1)

c)  $f(x) = x \cdot e^x$  ;  $f'(x) = 1 \cdot e^x + x \cdot e^x = \underbrace{e^x}_{\neq 0} (1+x)$

$\Rightarrow$  notw. Bed. Extrema  $f'(x) = 0 = \underbrace{e^x}_{\neq 0} (1+x) \Rightarrow x_E = -1$

$f''(x) = e^x + 1 \cdot e^x + x \cdot e^x = 2 \cdot e^x + x \cdot e^x = e^x (2+x)$

hinr. Bed.  $f''(-1) = 2 \cdot e^{-1} + (-1) \cdot e^{-1} = \frac{2}{e} - \frac{1}{e} = \frac{1}{e} > 0$

$\Rightarrow$  T(-1| -1 \cdot e^{-1}) = (-1| -\frac{1}{e})

d)  $f(x) = x^2 \cdot e^{0,5x}$  ;  $f'(x) = 2x \cdot e^{0,5x} + x^2 \cdot e^{0,5x} \cdot 0,5$

$f'(x) = x \cdot \underbrace{e^{0,5x}}_{\neq 0} (2 + 0,5x)$

notw. Bed. Extrema  $f'(x) = 0 \Rightarrow \underline{x_{E_1} = 0} \vee (2 + 0,5x) = 0 \Rightarrow \underline{x_{E_2} = -4}$

hinr. Bed  $f'(x) > 0$  für  $x < -4$  ; da  $x < 0$  ;  $e^{0,5x} > 0$  ;  $2 + 0,5x < 0$

$f'(x) < 0$  für  $-4 < x < 0$  ; da  $x < 0$  ;  $e^{0,5x} > 0$  ;  $2 + 0,5x > 0$

$f'(x) > 0$  für  $0 < x$  ; da  $0 < x$  ;  $e^{0,5x} > 0$  ;  $2 + 0,5x > 0$

$\Rightarrow$  H(-4|  $16e^{-2}$ ) = (-4|  $\frac{16}{e^2}$ ) ; T(0|0)