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Nr. 1) a) $V = \pi \cdot \int_{-1}^{2} \left(\sqrt{x+1}\right)^2 dx = \pi \left[\frac{x^2}{2} + x\right]_{-1}^{2}$

$= \pi \cdot \left(2 + 2 - \left\{\frac{1}{2} - 1\right\}\right) = \underline{\underline{\pi \cdot 4{,}5}}$

b) $V = \pi \cdot \int_{1}^{3} \left(\frac{1}{x}\right)^2 dx = \pi \cdot \int_{1}^{3} \frac{1}{x^2} dx = \pi \cdot \left[\frac{x^{-1}}{-1}\right]_{1}^{3} = \pi \cdot \left[-\frac{1}{x}\right]_{1}^{3}$

$= \pi \cdot \left(-\frac{1}{3} - \left\{-\frac{1}{1}\right\}\right) = \underline{\underline{\pi \cdot \frac{2}{3}}}$

c) $V = 2 \cdot \pi \cdot \int_{0}^{2} \left(x^2 - 4\right)^2 dx =$

Schnitt mit x-Achse
$x^2 - 4 = 0 \Rightarrow \underline{x_{1,2} = \pm 2}$

$V = 2 \cdot \pi \cdot \int_{0}^{2} \left(x^4 - 8x^2 + 16\right) dx = 2\pi \cdot \left[\frac{x^5}{5} - 8 \cdot \frac{x^3}{3} + 16x\right]_{0}^{2}$

$V = 2\pi \cdot \left(\frac{32}{5} - \frac{64}{3} + 32 - \{0\}\right) = \underline{\underline{2\pi \cdot \frac{256}{15} \approx 107{,}23}}$

Nr. 2) a) $V = \pi \cdot \int_{0}^{5} \left(2 \cdot e^{-0{,}4x}\right)^2 dx = \pi \cdot \int_{0}^{5} \left(4 \cdot e^{-0{,}8x}\right) dx$

$= \pi \cdot \left[4 \cdot e^{-0{,}8x} : \frac{1}{-0{,}8}\right]_{0}^{5} = \pi \cdot \left[-5 \cdot e^{-0{,}8x}\right]_{0}^{5}$

$= \pi \cdot \left(-5 \cdot e^{-4} - \{-5 \cdot e^{0}\}\right) = -\frac{5 \cdot \pi}{e^4} + 5 \cdot \pi = \underline{\underline{5\pi \cdot \left(-\frac{1}{e^4} + 1\right) \approx 15{,}42}}$

b) $V = \pi \cdot \int_{2}^{4} \left(0{,}5x + 1\right)^2 dx = \pi \int_{2}^{4} \left(\frac{1}{4}x^2 + x + 1\right) dx = \pi \left[\frac{1}{4} \cdot \frac{x^3}{3} + \frac{x^2}{2} + x\right]_{2}^{4}$

$= \pi \cdot \left(\frac{64}{12} + \frac{16}{2} + 4 - \left\{\frac{8}{12} + \frac{4}{2} + 2\right\}\right) = \underline{\underline{\pi \cdot \frac{38}{3} \approx 39{,}79}}$

c) $V = \pi \cdot \int_{2}^{5} \left(\frac{1}{(x-1)^2}\right)^2 dx = \pi \int_{2}^{5} \frac{1}{(x-1)^4} dx = \pi \int_{2}^{5} (x-1)^{-4} dx$

$V = \pi \cdot \left[\frac{(x-1)^{-3}}{-3}\right]_{2}^{5} = \pi \cdot \left[-\frac{1}{3 \cdot (x-1)^3}\right]_{2}^{5} = \pi \cdot \left(-\frac{1}{3 \cdot 64} - \left\{-\frac{1}{3} \cdot \frac{1}{1}\right\}\right)$

$V = \underline{\underline{\frac{21}{64} \cdot \pi \approx 1{,}03}}$

0. Fell