LS-Kursstufe Seite 102

Nr. 3a) $f(x) = 3x - \frac{1}{2}x^2$ Integrationsgrenzen $\hat{=}$ Nullstellen

$$f(x) = 0 = x\left(3 - \frac{1}{2}x\right) \Rightarrow x_1 = 0 \vee x_2 = 6$$

$$V = \pi \int_0^6 \left(3x - \frac{1}{2}x^2\right)^2 dx = \pi \cdot \int_0^6 \left(9x^2 - 3x^3 + \frac{1}{4}x^4\right) dx$$

$$V = \pi \cdot \left[9 \cdot \frac{x^3}{3} - 3 \cdot \frac{x^4}{4} + \frac{1}{4} \cdot \frac{x^5}{5}\right]_0^6 = \pi \cdot (648 - 972 + 388{,}8 - \{0\}) = \underline{\underline{64{,}8 \cdot \pi}}$$

3b) $f(x) = x(x+2) = 0 \Rightarrow x_1 = 0 \vee x_2 = -2$

$$V = \pi \cdot \int_{-2}^0 (x^2 + 2x)^2 dx = \pi \cdot \int_{-2}^0 (x^4 + 4x^3 + 4x^2) dx = \pi \left[\frac{x^5}{5} + 4 \cdot \frac{x^4}{4} + 4 \cdot \frac{x^3}{3}\right]_{-2}^0$$

$$V = \pi \cdot \left(0 - \left\{\frac{(-2)^5}{5} + \cancel{4} \cdot \frac{(-2)^4}{\cancel{4}} + 4 \cdot \frac{(-2)^3}{3}\right\}\right) = \pi \cdot \left(\frac{32}{5} - 16 + \frac{32}{3}\right) = \underline{\underline{\frac{16}{15} \cdot \pi}}$$

3c) $f(x) = x\sqrt{4-x} = 0 \Rightarrow x_1 = 0 \vee x = 4$

$$V = \pi \cdot \int_0^4 \left(x\sqrt{4-x}\right)^2 dx = \pi \int_0^4 \left(\sqrt{4x^2 - x^3}\right)^2 dx = \pi \int_0^4 (4x^2 - x^3) dx$$

$$V = \pi \cdot \left[4 \cdot \frac{x^3}{3} - \frac{x^4}{4}\right]_0^4 = \pi \cdot \left(4 \cdot \frac{4^3}{3} - \frac{4^4}{4} - \{0\}\right) = \underline{\underline{\pi \cdot \frac{64}{3}}}$$

3d) $f(x) = (x-1)(x+1) = 0 \Rightarrow x_1 = -1 \vee x_2 = +1$

$$V = \pi \cdot \int_{-1}^1 \left((x-1)(x+1)\right)^2 dx = \pi \cdot \int_{-1}^1 (x^2 - 1)^2 dx = \pi \int_{-1}^1 (x^4 - 2x^2 + 1) dx$$

$$V = \pi \cdot \left[\frac{x^5}{5} - 2 \cdot \frac{x^3}{3} + x\right]_{-1}^1 = \pi \cdot \left(\frac{1}{5} - \frac{2}{3} + 1 - \left\{-\frac{1}{5} + \frac{2}{3} - 1\right\}\right)$$

$$\underline{\underline{V = \pi \cdot \frac{16}{15}}}$$

O. Fell