Nr. 10)

a) $J(z) = \int_1^z e^{-x}\,dx = \left[e^{-x}\cdot\frac{1}{-1}\right]_1^z = \left[-e^{-x}\right]_1^z$

$\lim\limits_{z\to\infty}\left(-e^{-z} - \{-e^{-1}\}\right) = \lim\limits_{z\to\infty}\left(\underbrace{-\frac{1}{e^z}}_{\to 0} + \frac{1}{e}\right) = \underline{\underline{\frac{1}{e}}}$

$J(2) = \frac{-1}{e^2} + \frac{1}{e} \Rightarrow p = \frac{-\frac{1}{e^2}+\frac{1}{e}}{\frac{1}{e}} = -\frac{e}{e^2}+1 = -\frac{1}{e}+1 \approx \underline{\underline{0{,}632}}$

$63{,}2\%$

$J(5) = \frac{-1}{e^5} + \frac{1}{e} \Rightarrow p = \frac{-\frac{1}{e^5}+\frac{1}{e}}{\frac{1}{e}} = -\frac{e}{e^5}+1 = -\frac{1}{e^4}+1 \approx \underline{\underline{0{,}982}}$

$98{,}2\%$

$J(n) = -\frac{1}{e^n} + \frac{1}{e} \Rightarrow \underline{\underline{p = -\frac{1}{e^{n-1}}+1}}\ ;\ n \in \mathbb{N}$

Für $n = 10$

$J(10) = -\frac{1}{e^{10}} + \frac{1}{e} \Rightarrow p = -\frac{1}{e^9}+1 \approx \underline{\underline{0{,}99988}} = \underline{\underline{99{,}988\%}}$

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b) $J(z) = \int_1^z x^{-2}\,dx = \int_1^z \frac{1}{x^2}\,dx = \left[\frac{x^{-1}}{-1}\right]_1^z = \left[-\frac{1}{x}\right]_1^z$

$J(z) = -\frac{1}{z} - \left\{-\frac{1}{1}\right\} = -\frac{1}{z}+1$

$\lim\limits_{z\to\infty}\left(\underbrace{-\frac{1}{z}}_{\to 0}+1\right) = 1$

$J(2) = -\frac{1}{2}+1 \Rightarrow p = \frac{-\frac{1}{2}+1}{1} = \underline{\underline{0{,}5 = 50\%}}$

$J(5) = -\frac{1}{5}+1 \Rightarrow p = \frac{-\frac{1}{5}+1}{1} = \frac{4}{5} = \underline{\underline{0{,}8 = 80\%}}$

$J(n) = -\frac{1}{n}+1 \Rightarrow \underline{\underline{p = \frac{-\frac{1}{n}+1}{1} = \frac{n-1}{n}}}\ ;\ n \in \mathbb{N}$

Für $n = 10$

$J(10) = -\frac{1}{10}+1 \Rightarrow p = \frac{9}{10} = \underline{\underline{0{,}9 = 90\%}}$

O. Fell