Nr. 9)

a) I. $J(z) = \int_1^z \frac{1}{x^3}\,dx = \left[\frac{x^{-2}}{-2}\right]_1^z = \left[\frac{-1}{2x^2}\right]_1^z = -\frac{1}{2z^2} - \left\{-\frac{1}{2}\right\}$

$\lim\limits_{z\to\infty}\left(\underbrace{-\frac{1}{2z^2}}_{\to 0} + \frac{1}{2}\right) = \underline{\underline{\frac{1}{2}}}$ Das uneigentliche Integral existiert

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II $J(z) = \int_1^z \frac{1}{x^2}\,dx = \left[\frac{x^{-1}}{-1}\right]_1^z = \left[\frac{-1}{x}\right]_1^z = -\frac{1}{z} - \left\{\frac{-1}{1}\right\}$

$\lim\limits_{z\to\infty}\left(\underbrace{-\frac{1}{z}}_{\to 0} + 1\right) = \underline{\underline{1}}$ Das uneigentliche Integral existiert

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III $J(z) = \int_1^z \frac{1}{\sqrt{x}}\,dx = \int_1^z x^{-\frac{1}{2}}\,dx = \left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^z = \left[2\sqrt{x}\right]_1^z$

$J(z) = 2\sqrt{z} - \{2\sqrt{1}\} = 2\sqrt{z} - 2$; $\underline{\underline{J(z)\to\infty}}$ für $z\to\infty$

Das uneigentliche Integral existiert nicht

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b) I. $J(z) = \int_z^1 \frac{1}{x^3}\,dx = \left[\frac{x^{-2}}{-2}\right]_z^1 = \left[\frac{-1}{2x^2}\right]_z^1 = -\frac{1}{2\cdot 1} - \left\{\frac{-1}{2z^2}\right\}$

$J(z) = -\frac{1}{2} + \frac{1}{2z^2}$; $\underline{\underline{J(z)\to\infty}}$ $\underline{\underline{\text{für } z\to 0}}$ Das uneigentliche Integral existiert <u>nicht</u>.

II $J(z) = \int_z^1 \frac{1}{x^2}\,dx = \left[\frac{x^{-1}}{-1}\right]_z^1 = \left[\frac{-1}{x}\right]_z^1 = -\frac{1}{1} - \left\{\frac{-1}{z}\right\} = \frac{1}{z} - 1$

$\underline{\underline{J(z)\to\infty \text{ für } z\to 0}}$ Das uneigentliche Integral existiert <u>nicht</u>.

III $J(z) = \int_z^1 \frac{1}{\sqrt{x}}\,dx = \int_z^1 x^{-\frac{1}{2}}\,dx = \left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_z^1 = \left[2\sqrt{x}\right]_z^1$

$J(z) = 2\cdot\sqrt{1} - \{2\cdot\sqrt{z}\} = 2 - 2\sqrt{z}$

$\lim\limits_{z\to 0}\left(2 - \underbrace{2\sqrt{z}}_{\to 0}\right) = \underline{\underline{2}}$ Das uneigentliche Integral <u>existiert</u>

O. Fell