Nr. 10) $f(x) = -\frac{1}{16}x^4 + \frac{1}{2}x^2 - 1$

a) Höhe der Schale $f(0) = -1 \Rightarrow |f(0)| = \underline{\underline{1}}$

Die Schale ist $\underline{\underline{1\,dm}}$ hoch

Radius der Schale $\hat{=}$ Abstand Nullstellen vom Origo

$f(x) = 0 = -\frac{1}{16}x^4 + \frac{1}{2}x^2 - 1 \quad |\ \text{Sub}: x^2 = u$

$-\frac{1}{16}u^2 + \frac{1}{2}u - 1 = 0 \Rightarrow u_{1,2} = \dfrac{-\frac{1}{2} \pm \sqrt{\frac{1}{4} - 4\cdot(-\frac{1}{16})\cdot(-1)}}{2\cdot(-\frac{1}{16})}$

$u_1 = \dfrac{-\frac{1}{2} \pm 0}{-\frac{1}{8}} = \frac{1}{2}\cdot\frac{8}{1} = \underline{\underline{4}} \qquad u_2 = u_1 = 4$

Rück. Sub. $x^2 = 4 \Rightarrow x_{1,2} = \pm\sqrt{4} = (\pm)\underline{\underline{2}}$

Der Radius R beträgt $\underline{\underline{2\,dm}}$

b) $f(0{,}9) = -\frac{1}{16}\cdot 0{,}9^4 + \frac{1}{2}\cdot 0{,}9^2 - 1 \approx \underline{\underline{-0{,}636}}$

$|f(0{,}9)| = \underline{\underline{0{,}636\,dm}} > h_D = 0{,}6\,dm \Rightarrow$ Dose passt ganz in die Schale.

c) Gesucht stelle mit Funktioswert $-0{,}5$

$f(x) = -0{,}5 = -\frac{1}{16}x^4 + \frac{1}{2}x^2 - 1 \quad |+0{,}5$

$-\frac{1}{16}x^4 + \frac{1}{2}x^2 - 0{,}5 = 0 \quad |\ \text{Sub}\ x^2 = u \Rightarrow -\frac{1}{16}u^2 + \frac{1}{2}u - 0{,}5 = 0$

$u_{1,2} = \dfrac{-\frac{1}{2} \pm \sqrt{\frac{1}{4} - 4\cdot(-\frac{1}{16})\cdot(-0{,}5)}}{2\cdot(-\frac{1}{16})} = \dfrac{-\frac{1}{2} \pm \sqrt{\frac{1}{8}}}{-\frac{1}{8}} = \dfrac{-\frac{1}{2} \pm \frac{1}{2}\cdot\frac{1}{\sqrt{2}}}{-\frac{1}{8}}$

$u_{1,2} = \dfrac{-\frac{1}{2} \pm \frac{1}{2}\cdot\frac{\sqrt{2}}{2}}{-\frac{1}{8}} = 8\cdot\left(\frac{1}{2} \pm \frac{1}{4}\sqrt{2}\right) = 4 \pm 2\sqrt{2}$

Rück. Sub. $x^2 = \pm\sqrt{4 + 2\sqrt{2}} \Rightarrow x_{1,2} = (\pm)2{,}61 > R = 2$

$\Rightarrow x_1 = 2{,}61\,dm$ nicht in der Schale

Rück. Sub: $x^2 = \pm\sqrt{4 - 2\sqrt{2}} \Rightarrow x_{3,4} = (\pm)\,1{,}082 < R = 2$

Der kleine Radius $\underline{\underline{r = 1{,}082\,dm}}$

O. Fell