

Nr. 8) a)  $f(x) = g(x) \Leftrightarrow 2e^x = e^{2x} - 3 \quad | -2e^x$

$\Rightarrow e^{2x} - 2e^x - 3 = 0 \quad | \text{Sub: } e^x = u$

$u^2 - 2u - 3 = 0 \Rightarrow u_{1,2} = \frac{+2 \pm \sqrt{4 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1} = \frac{+2 \pm 4}{2}$

$u_1 = 3$

$\vee \quad u_3 = -1$

Rück  $e^x = 3 \quad | \ln$

Sub  $\Rightarrow x = \ln(3)$

$e^x = -1 \quad \nexists$

keine Lösung

$\Rightarrow$  Schnitstelle  $x_1 = \ln(3)$

b)  $f(x) = e^{3x} = e^x + 2 \cdot e^{-x} = g(x)$

$e^{3x} - e^x - 2e^{-x} = 0 \quad | \text{Sub: } e^x = u \text{ (I)}$

$u^3 - u - \frac{2}{u} = 0 \quad | \cdot u \Rightarrow u^4 - u^2 - 2 = 0 \quad | \text{Sub: } u^2 = z \text{ (II)}$

$z^2 - z - 2 = 0 \Rightarrow z_{1,2} = \frac{+1 \pm \sqrt{1 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1} = \frac{+1 \pm 3}{2}$

$z_1 = 2 \quad \text{Rück Sub. II } u_{1,2} = \pm \sqrt{2}$

$z_2 = -1 \quad \text{Rück Sub. II } u_{3,4} = \nexists \quad u^2 = -1 \text{ keine Lösung}$

Rück: Sub I  $e^x = +\sqrt{2} \quad | \ln \Rightarrow x_1 = \ln(\sqrt{2}) = \ln(2^{\frac{1}{2}})$

$(e^x = -\sqrt{2} \quad \nexists)$

$x_1 = \frac{1}{2} \ln(2)$

c)  $f(x) = e^x = \ln(x^2 + 1) \cdot e^x = g(x) \quad | -e^x$

$0 = \ln(x^2 + 1) \cdot e^x - e^x = \overset{\neq 0}{e^x} (\ln(x^2 + 1) - 1) = 0$

$\Rightarrow \ln(x^2 + 1) - 1 = 0 \Rightarrow \ln(x^2 + 1) = 1 \quad | e^{\uparrow} \Rightarrow x^2 + 1 = e^1$

$\Rightarrow x^2 = e - 1 \quad | \sqrt{\phantom{x}} \Rightarrow \underline{\underline{x_{1,2} = \pm \sqrt{e-1}}}$

Nr. 8) d)  $f(x) = e^x \cdot (\ln(x) + 3) = e^x \cdot (5 \cdot \ln(x) - 1) = g(x) \quad | -g(x)$   
 $0 = e^x (\ln(x) + 3) - e^x \cdot (5 \cdot \ln(x) - 1) = \underbrace{e^x}_{\neq 0} \cdot (\ln(x) + 3 - 5 \ln(x) + 1)$   
 $\Rightarrow -4 \ln(x) + 4 = 0 \quad | -4 \Rightarrow -4 \ln(x) = -4 \quad | :(-4)$   
 $\Rightarrow \ln(x) = 1 \quad | e^{\uparrow} \Rightarrow \underline{\underline{x_1 = e^1 = e}}$

Nr. 9) a)  $f(x) = (x-3) \cdot (x+2)$   
 $g(x) = (x-3)^2 \cdot (x+2)$

b)  $f(x) = (x-e) \cdot (x-(e+1))$   
 $g(x) = (\ln(x) - 1) \cdot (x - (e+1))$

c)  $f(x) = x(x - e^{-1}) = x \cdot (x - \frac{1}{e})$   
 $g(x) = x^2 \cdot (x - \frac{1}{e}) = x^3 - \frac{x^2}{e}$