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Nr. 10) $f_t(x) = x^3 - 12t^2x \quad ; \quad t \in \mathbb{R}^+$

a) Extrema notw. Bed $f_t'(x) = 3x^2 - 12t^2 = 0 \quad | +12t^2$

$3x^2 = 12t^2 \quad |:3 \Rightarrow x^2 = 4t^2 \Rightarrow x_{1,2} = \pm\sqrt{4t^2} = \pm 2t$

hinr. Bed $f_t''(x) = 6x \quad ; \quad f_t''(-2t) = 6\cdot(-2t) = -12t < 0 \; ; t > 0$

$f_t''(+2t) = 6\cdot(2t) = 12t > 0 \Rightarrow H(-2t \mid f_t(-2t)) \; ; \; T(+2t \mid f_t(+2t))$

$f_t(-2t) = (-2t)^3 - 12\cdot t^2\cdot(-2t) = -8t^3 + 24t^3 = 16t^3 \quad ; \quad t > 0$

$f_t(+2t) = (2t)^3 - 12\cdot t^2\cdot(2t) = 8t^3 - 24t^3 = -16t^3 \quad ; \quad t > 0$

$\underline{\underline{H(-2t \mid 16t^3)}} \quad ; \quad \underline{\underline{T(+2t \mid -16t^3)}}$

b) Ortskurve Hochpunkt: $x = -2t \Rightarrow t = -\frac{x}{2}$; einsetzen in $16t^3$

$\underline{\underline{y = 16\cdot\left(-\frac{x}{2}\right)^3 = -2x^3}}$

Ortskurve Tiefpunkte: $x = 2t \Rightarrow t = \frac{x}{2}$; einsetzen in $-16t^3$

$\underline{\underline{y = -16\cdot\left(\frac{x}{2}\right)^3 = -2x^3}}$ Alle Hochpunkte liegen im 2. Quadranten
Alle Tiefpunkte liegen im 4. Quadranten

c) Gemeinsame Punkte: $t_1 \neq t_2$

$x^3 - 12t_1^2x = x^3 - 12t_2^2x \quad | -x^3 \Rightarrow -12t_1^2x = -12t_2^2x \quad | +12t_2^2x$

$-12t_1^2x + 12t_2^2x = 0 \Rightarrow 12x\cdot(\underbrace{-t_1^2 + t_2^2}_{\neq 0}) = 0 \Rightarrow \underline{\underline{x_3 = 0}}$

einziger gemeinsamer Punkt: $\underline{\underline{S_G(0|0)}}$

d) $f_t(x) = x^3 - 12t^2\cdot x = 0 \quad ; \quad$ für $x \geq 0$

$x\cdot(x^2 - 12t^2) = 0 \Rightarrow \underline{\underline{x_4 = 0}} \vee x_{5(6)} = \underset{(-)}{+}\sqrt{12t^2} = \underline{\underline{+t\cdot 2\cdot\sqrt{3}}}$

$A_t = \left|\int_0^{t\cdot 2\cdot\sqrt{3}} (x^3 - 12t^2\cdot x)\,dx\right| = \left|\left[\frac{x^4}{4} - 12t^2\cdot\frac{x^2}{2}\right]_0^{t\cdot 2\cdot\sqrt{3}}\right| = \left|\left[\frac{1}{4}x^4 - 6t^2x^2\right]_0^{t\cdot 2\cdot\sqrt{3}}\right|$

$= \left|\frac{1}{4}t^4\cdot 16\cdot 9 - 6\cdot t^2\cdot t^2\cdot 4\cdot 3 - \{0\}\right| = |-36t^4| = \underline{\underline{36t^4}}$

e) $36\cdot t^4 = 2{,}25 \quad |:36 \Rightarrow t^4 = \frac{1}{16} \quad |\sqrt[4]{\;} \Rightarrow \underline{\underline{t = \frac{1}{2}}}$

f) $f_t'(0) = -12t^2 = -1 \Rightarrow t^2 = \frac{1}{12} \Rightarrow \underline{\underline{t = \underset{(-)}{+}\sqrt{\frac{1}{12}}}} = \sqrt{\frac{1}{4\cdot 3}} = \frac{1}{2\cdot\sqrt{3}} = \frac{\sqrt{3}}{2\cdot 3} = \underline{\underline{\frac{\sqrt{3}}{6}}}$

O. Fell