Nr. 4.) $f_k(x) = x \cdot e^{-kx}$; $k > 0$

$$f_k'(x) = 1 \cdot e^{-kx} + x \cdot e^{-kx} \cdot (-k) = \underbrace{e^{-kx}}_{>0} \cdot (1 - kx) = 0$$

Extrema notw. Bed. $f_k'(x) = 0 \Rightarrow 1 - kx = 0 \Rightarrow x_E = \frac{1}{k}$

hinr. Bed. $f_k'(x) > 0$ für $x < \frac{1}{k}$ weil $e^{-kx} > 0 \wedge 1 - kx > 0$

$f_k'(x) < 0$ für $\frac{1}{k} < x$ weil $e^{-kx} > 0 \wedge 1 - kx < 0$ } VZW von $f_k'(x)$

$$\Rightarrow H\left(\frac{1}{k} \middle| \frac{1}{k} \cdot e^{-k \cdot \frac{1}{k}}\right) = \left(\frac{1}{k} \middle| \frac{1}{k} \cdot e^{-1}\right) = \left(\frac{1}{k} \middle| \frac{1}{k \cdot e}\right)$$

Wendepunkt notw. Bed.

$$f_k''(x) = e^{-kx} \cdot (-k) \cdot (1 - kx) + e^{-kx} \cdot (-k)$$

$$= e^{-kx} \cdot (-k + k^2 x - k) = \underbrace{e^{-kx}}_{>0} \cdot (k^2 x - 2k) = 0$$

Nullprodukt: $k^2 x - 2k = 0 \mid +2k \Rightarrow k^2 x = 2k \mid : k^2 \Rightarrow x_w = \frac{2k}{k^2} = \frac{2}{k}$

hinr. Bed. $k^2 x - 2k$ wechselt an der Stelle $x_w = \frac{2}{k}$ das Vorzeichen $\Rightarrow f_k''(x)$ hat einen VZW an der Stelle $x_w = \frac{2}{k}$

$$\Rightarrow W\left(\frac{2}{k} \middle| \frac{2}{k} \cdot e^{-k \cdot \frac{2}{k}}\right) = \left(\frac{2}{k} \middle| \frac{2}{k} \cdot e^{-2}\right) = \left(\frac{2}{k} \middle| \frac{2}{k \cdot e^2}\right)$$

O. Fell