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Nr. 16) $f_t(x) = t\cdot\cos(x)$ mit $-\frac{\pi}{2} \le x \le +\frac{\pi}{2}$; $t>0$

Nullstellen von f ung g sind gleich $N_1(-\frac{\pi}{2}|0)$ $N_2(+\frac{\pi}{2}|0)$

$$g_a(x) = a\cdot\left(x+\frac{\pi}{2}\right)\cdot\left(x-\frac{\pi}{2}\right) = a\cdot\left(x^2-\left(\frac{\pi}{2}\right)^2\right)$$

Funktionen 2. Grades sind durch die Nullstellen bis auf einen Faktor a bestimmt

Schnitt soll $\perp$ sein!

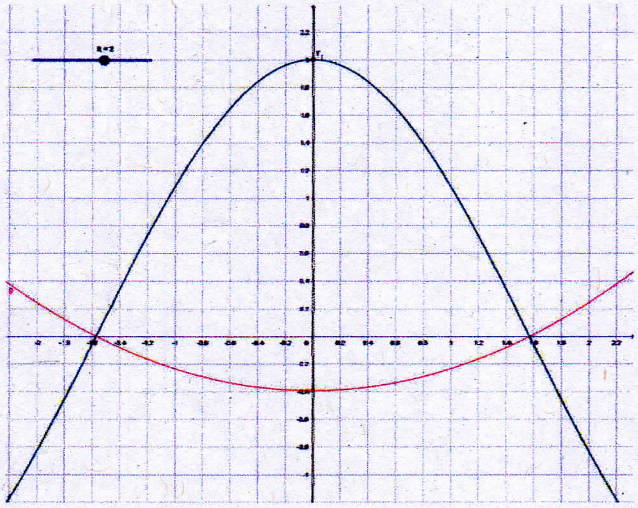
$$\Rightarrow f_t'\left(\frac{\pi}{2}\right)\cdot g_a'\left(\frac{\pi}{2}\right) = -1 \quad ; \quad f_t'(x) = t\cdot(-\sin(x)) = -t\sin(x)$$

$$g_a'(x) = a\cdot 2x$$

$$\Rightarrow -t\cdot\sin\left(\frac{\pi}{2}\right)\cdot a\cdot 2\cdot\frac{\pi}{2} = -1$$

$$-t\cdot 1\cdot a\cdot\pi = -1 \;\Big|:(-t\cdot\pi) \Rightarrow \underline{\underline{a = \frac{1}{t\cdot\pi}}} \Rightarrow g_t(x) = \frac{1}{t\cdot\pi}\cdot\left(x+\frac{\pi}{2}\right)\left(x-\frac{\pi}{2}\right)$$

$$g_t(x) = \frac{1}{t\cdot\pi}\cdot\left(x^2-\left(\frac{\pi}{2}\right)^2\right)$$



$$\underline{\underline{g_t(x)}} = \frac{x^2}{t\cdot\pi} - \frac{\pi^2}{4}\cdot\frac{1}{t\cdot\pi} = \underline{\underline{\frac{x^2}{t\cdot\pi} - \frac{\pi}{4\cdot t}}}$$

b) $$A(t) = 2\int_0^{\frac{\pi}{2}}\left(f_t(x) - g_t(x)\right)\cdot dx$$

$$A(t) = 2\cdot\int_0^{\frac{\pi}{2}}\left(t\cdot\cos(x) - \frac{x^2}{t\cdot\pi} + \frac{\pi}{4t}\right)dx$$

$$\Rightarrow A(t) = 2\cdot\left[t\cdot\sin(x) - \frac{x^3}{3\pi\cdot t} + \frac{\pi\cdot x}{4\cdot t}\right]_0^{\frac{\pi}{2}} = 2\cdot\left[t\cdot\sin\left(\frac{\pi}{2}\right) - \frac{\left(\frac{\pi}{2}\right)^3}{3\pi\cdot t} + \frac{\pi\cdot\frac{\pi}{2}}{4\cdot t} - 0\right]$$

$$A(t) = 2\cdot\left[t\cdot 1 - \frac{\pi^3}{8\cdot 3\cdot\pi\cdot t} + \frac{\pi^2}{2\cdot 4\cdot t}\right] = 2\cdot\left[t - \frac{\pi^2}{24\cdot t} + \frac{\pi^2}{8t}\right]$$

$$\underline{\underline{A(t)}} = 2\cdot\left[t + \frac{2\pi^2}{24t}\right] = \underline{\underline{2\cdot t + \frac{\pi^2}{6t}}}$$

Extrema von $A(t) \Rightarrow A'(t) = 2 - \frac{\pi^2}{6}\cdot t^{-2} = 2 - \frac{\pi^2}{6\cdot t^2} = 0$ notw. Bed

$$\Rightarrow 2 = \frac{\pi^2}{6\cdot t^2} \;\Big|\cdot 6\cdot t^2 \Rightarrow 12\cdot t^2 = \pi^2 \;\Big|:12 \;\Big|\sqrt{\;} \Rightarrow t_{1,2} = \overset{+}{(-)}\sqrt{\frac{\pi^2}{12}} \;;\; t>0$$

$\Rightarrow \underline{\underline{t = \frac{\pi}{\sqrt{12}}}}$; Hinr. Bed $A''(t) = +\frac{1}{3}\frac{\pi^2}{t^3} > 0$ für $t>0 \Rightarrow$ Minimum

$$\Rightarrow \underline{\underline{A_{min}}} = 2\cdot\frac{\pi}{\sqrt{12}} + \frac{\pi^2}{6\cdot\frac{\pi}{\sqrt{12}}} = \frac{2\cdot\pi}{\sqrt{12}} + \frac{\pi\sqrt{12}}{6} = \frac{2\pi}{2\sqrt{3}} + \frac{\pi\cdot 2\sqrt{3}}{6} = \underline{\underline{\frac{\pi}{\sqrt{3}} + \frac{\pi}{3}\sqrt{3}}}$$

O. Fell