

$$1.) a) \quad 2x_1 - 3x_2 - 5x_3 = -1$$

$$2x_2 + 1x_3 = 0$$

$$3x_3 = 6 \quad | :3$$

$$\underline{x_3 = \frac{6}{3} = 2}$$

$$2x_2 + 1 \cdot 2 = 0 \Rightarrow 2x_2 = -2 \quad | :2 \Rightarrow \underline{x_2 = -1}$$

$$2x_1 - 3 \cdot (-1) - 5 \cdot 2 = -1 \Rightarrow 2x_1 = 6 \Rightarrow \underline{x_1 = 3}$$

$$\underline{\underline{L = \{(3 | -1 | 2)\}}}$$

$$b) \quad 3x_1 + 8x_2 - 3x_3 = 5$$

$$4x_2 + x_3 = 1$$

$$-5x_3 = 10 \quad | :(-5)$$

$$\underline{x_3 = -2}$$

$$4x_2 + (-2) = 1 \Rightarrow 4x_2 = 3 \quad | :4 \Rightarrow \underline{x_2 = \frac{3}{4}}$$

$$3x_1 + 8 \cdot \frac{3}{4} - 3 \cdot (-2) = 5 \Rightarrow 3x_1 = -7 \quad | :3 \Rightarrow \underline{x_1 = -\frac{7}{3}}$$

$$\underline{\underline{L = \left\{ \left(-\frac{7}{3} \mid \frac{3}{4} \mid -2 \right) \right\}}}$$

$$c) \quad 3x_1 + 4x_2 + 6x_3 = 5$$

$$17x_2 + 24x_3 = 16$$

$$2x_3 = 7 \quad | :2$$

$$\underline{x_3 = \frac{7}{2}}$$

$$17x_2 + 24 \cdot \frac{7}{2} = 16 \Rightarrow 17x_2 = -68 \quad | :17 \Rightarrow \underline{x_2 = -4}$$

$$3x_1 + 4 \cdot (-4) + 6 \cdot \frac{7}{2} = 5 \Rightarrow 3x_1 = 0 \Rightarrow x_1 = 0$$

$$\underline{\underline{L = \left\{ \left(0 \mid -4 \mid \frac{7}{2} \right) \right\}}}$$

2.) I $x_1 + x_2 + 2x_3 = 14$

a) II $7x_2 + x_3 = 0$ | $\cdot 1$
 III $7x_2 - x_3 = -14$ | $\cdot 1$ } alternative Schreibweise für
 $\underline{\text{III}_a = \text{II} + \text{III}}$

$x_1 + x_2 + 2x_3 = 14$

$7x_2 + x_3 = 0$

$\underline{\text{III}_a} \quad 14x_2 = -14 \Rightarrow \underline{x_2 = -1}$

$7 \cdot (-1) + x_3 = 0 \Rightarrow x_3 = 7$

$x_1 + (-1) + 2 \cdot 7 = 14 \Rightarrow \underline{x_1 = 1}$

$\underline{\underline{L = \{(1 | -1 | 7)\}}}$

b) I $2x_1 - 6x_2 + x_3 = -1$ | $\cdot 1$

II $2x_1 + x_2 + 3x_3 = 0$ | $\cdot (-1)$ } $\underline{\text{II}_0 = \text{I} + (-1) \cdot \text{II}}$

III $2x_3 = -6$

$2x_1 - 6x_2 + x_3 = -1$

$\underline{\text{II}_0} \quad -7x_2 - 2x_3 = -1$

$2x_3 = -6 \Rightarrow \underline{x_3 = -3}$

$-7x_2 - 2 \cdot (-3) = -1 \Rightarrow -7x_2 = -7 \Rightarrow \underline{x_2 = 1}$

$2x_1 - 6 \cdot 1 + (-3) = -1 \Rightarrow 2x_1 = 8 \Rightarrow \underline{x_1 = 4} \quad \underline{\underline{L = \{(4 | 1 | -3)\}}}$

c) I $4x_1 + 4x_2 + 6x_3 = 8$ | $\cdot 1$

II $6x_2 + x_3 = -2$ | $\cdot 1$ } $\underline{\text{III}_a = \text{I} + (-1) \cdot \text{II}}$

III $4x_1 - 2x_2 + 7x_3 = 6$ | $\cdot (-1)$

$\text{I}_0 \quad 4x_1 + 4x_2 + 6x_3 = 8$

$\underline{\text{II}_a} \quad 6x_2 + x_3 = -2$ | $\cdot 1$ } $\underline{\text{III}_b = \text{II}_a + \text{III}_a}$

$\underline{\text{III}_a} \quad 6x_2 - x_3 = 2$ | $\cdot 1$

$4x_1 + 4x_2 + 6x_3 = 8$

$6x_2 + x_3 = -2$

$\underline{\text{III}_b} \quad 12x_2 = 0 \Rightarrow \underline{x_2 = 0}$

$6 \cdot 0 + x_3 = -2 \Rightarrow \underline{x_3 = -2}$

$4x_1 + 4 \cdot 0 + 6 \cdot (-2) = 8 \Rightarrow 4x_1 = 20 \Rightarrow \underline{x_1 = 5} \quad \underline{\underline{L = \{(5 | 0 | -2)\}}}$