

Nr. 3) a)  $\begin{array}{l} \text{I} \quad x_1 + x_2 + 4x_3 = 0 \\ \text{II} \quad 2x_1 - x_2 - x_3 = 3 \\ \text{III} \quad -3x_1 + 4x_2 - 5x_3 = -7 \end{array}$   $\left. \begin{array}{l} \cdot 2 \\ \cdot (-1) \\ \cdot 1 \end{array} \right\} \rightarrow \text{alternative Schreibweise f\u00fcr } \underline{\text{II}}_a = 2 \cdot \text{I} + (-1) \cdot \text{II}$

$\underline{\text{III}}_a = 3 \cdot \text{I} + 1 \cdot \text{III}$

$$\begin{array}{l} x_1 + x_2 + 4x_3 = 0 \\ \underline{\text{II}}_a \quad 3x_2 + 9x_3 = -3 \quad | \cdot 7 \\ \underline{\text{III}}_a \quad 7x_2 + 7x_3 = -7 \quad | \cdot (-3) \end{array} \left. \right\} \underline{\text{III}}_b = 7 \cdot \underline{\text{II}}_a - 3 \cdot \underline{\text{III}}_a$$

$$\begin{aligned} x_1 + x_2 + 4x_3 = 0 &\Rightarrow x_1 + (-1) + 4 \cdot 0 = 0 \Rightarrow \underline{x_1 = 1} \\ 3x_2 + 9x_3 = -3 &\Rightarrow 3x_2 + 9 \cdot 0 = -3 \Rightarrow \underline{x_2 = -1} \\ +42x_3 = 0 &\Rightarrow \underline{x_3 = 0} \end{aligned}$$

$\mathbb{L} = \{(1 | -1 | 0)\}$

b)  $\begin{array}{l} 2x_1 - x_2 - x_3 = 2 \quad | \cdot 2 \\ 4x_1 \quad \quad + 6x_3 = 2 \\ -3x_1 + 2x_2 - x_3 = 1 \quad | \cdot 1 \end{array}$

$$\begin{array}{l} 2x_1 - x_2 - x_3 = 2 \\ 4x_1 \quad \quad + 6x_3 = 2 \quad | \cdot 1 \\ 1 \cdot x_1 \quad \quad - 3x_3 = 5 \quad | \cdot 2 \end{array}$$

$$\begin{aligned} 2x_1 - x_2 - x_3 = 2 &\Rightarrow 2 \cdot 2 - x_2 - (-1) = 2 \Rightarrow -x_2 = -3 \Rightarrow \underline{x_2 = 3} \\ 4x_1 \quad \quad + 6x_3 = 2 &\Rightarrow 4 \cdot 2 + 6x_3 = 2 \Rightarrow 6x_3 = -6 \Rightarrow \underline{x_3 = -1} \\ 6x_1 \quad \quad \quad = 12 &\Rightarrow \underline{x_1 = 2} \quad \mathbb{L} = \{(2 | 3 | -1)\} \end{aligned}$$

c)  $\begin{array}{l} x_2 + x_3 = 3 \quad | \cdot 1 \\ x_1 + 2x_2 = 7 \\ 2x_1 \quad \quad - x_3 = 8 \quad | \cdot 1 \end{array}$

$$\begin{array}{l} x_2 + x_3 = 3 \\ x_1 + 2x_2 = 7 \quad | \cdot 2 \\ 2x_1 + x_2 = 11 \quad | \cdot (-1) \end{array}$$

$$\begin{aligned} x_2 + x_3 = 3 &\Rightarrow 1 + x_3 = 3 \Rightarrow \underline{x_3 = 2} \\ x_1 + 2x_2 = 7 &\Rightarrow x_1 + 2 \cdot 1 = 7 \Rightarrow \underline{x_1 = 5} \\ 3x_2 = 3 &\Rightarrow \underline{x_2 = 1} \end{aligned}$$

$\mathbb{L} = \{(5 | 1 | 2)\}$