

Nr. 5) a) $\left(\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & 3 & 10 \end{array} \right) \begin{array}{l} \cdot 2 \\ \cdot (-1) \end{array}$

$$\begin{pmatrix} 1 & 1 & | & 2 \\ 0 & -1 & | & -6 \end{pmatrix} \Rightarrow \begin{array}{l} x_1 + 6 = 2 \Rightarrow x_1 = -4 \\ x_2 = 6 \end{array}$$

$\mathbb{L} = \{(-4 | 6)\}$

b) $\left(\begin{array}{cc|c} 2 & -1 & -1 \\ 4 & -3 & -4 \end{array} \right) \begin{array}{l} \cdot 2 \\ \cdot (-1) \end{array}$

$$\begin{pmatrix} 2 & -1 & | & -1 \\ 0 & 1 & | & 2 \end{pmatrix} \Rightarrow \begin{array}{l} 2x_1 - 2 = -1 \Rightarrow x_1 = \frac{1}{2} \\ x_2 = 2 \end{array}$$

$\mathbb{L} = \{(0,5 | 2)\}$

c) $\left(\begin{array}{cc|c} 3 & 6 & 0 \\ 6 & 6 & -1 \end{array} \right) \begin{array}{l} \cdot 2 \\ \cdot (-1) \end{array}$

$$\begin{pmatrix} 3 & 6 & | & 0 \\ 0 & 6 & | & 1 \end{pmatrix} \Rightarrow \begin{array}{l} 3x_1 + 6 \cdot \frac{1}{6} = 0 \Rightarrow x_1 = -\frac{1}{3} \\ x_2 = \frac{1}{6} \end{array}$$

$\mathbb{L} = \left\{ \left(-\frac{1}{3} \mid \frac{1}{6} \right) \right\}$

Nr. 6) a) $\left(\begin{array}{ccc|c} 1 & 5 & -1 & 1 \\ -1 & -4 & 1 & 0 \\ 1 & 10 & -3 & 4 \end{array} \right) \begin{array}{l} \cdot 1 \\ \cdot 1 \\ \cdot (-1) \end{array}$

$$\begin{pmatrix} 1 & 5 & -1 & | & 1 \\ 0 & 1 & 0 & | & 1 \\ 0 & -5 & 2 & | & -3 \end{pmatrix} \Rightarrow \begin{array}{l} x_1 + 5 \cdot 1 - 1 \cdot 1 = 1 \Rightarrow x_1 = -3 \\ x_2 = 1 \\ -5 \cdot 1 + 2x_3 = -3 \Rightarrow x_3 = 1 \end{array}$$

$\mathbb{L} = \{(-3 | 1 | 1)\}$

b) $\left(\begin{array}{ccc|c} 2 & 4 & 0 & -12 \\ -3 & -4 & 2 & 10 \\ 7 & 1 & 0 & 23 \end{array} \right) \begin{array}{l} \cdot (-1) \\ \\ \cdot (4) \end{array}$

$$\begin{pmatrix} -2 & 4 & 0 & | & -12 \\ -3 & -4 & 2 & | & 10 \\ 26 & 0 & 0 & | & 104 \end{pmatrix} \Rightarrow \begin{array}{l} 2 \cdot 4 + 4x_2 = -12 \Rightarrow x_2 = -5 \\ -3 \cdot 4 - 4 \cdot (-5) + 2x_3 = 10 \Rightarrow 2x_3 = 2 \Rightarrow x_3 = 1 \\ x_4 = \frac{104}{26} = 4 \end{array}$$

$\mathbb{L} = \{(4 | -5 | 1)\}$

c) $\left(\begin{array}{ccc|c} 12 & 2 & 4 & 6 \\ -5 & -4 & 8 & -4 \\ 2 & 1 & -2 & 0 \end{array} \right) \begin{array}{l} \cdot 2 \\ \cdot (-1) \\ \cdot (2) \end{array}$ $\left(\begin{array}{ccc|c} 12 & 2 & 4 & 6 \\ 29 & 8 & 0 & 16 \\ -3 & 0 & 0 & 4 \end{array} \right) \Rightarrow x_1 = -\frac{4}{3}$

$$\begin{pmatrix} 12 & 2 & 4 & | & 6 \\ 29 & 8 & 0 & | & 16 \\ 16 & 4 & 0 & | & 6 \end{pmatrix} \begin{array}{l} \cdot 1 \\ \cdot (-2) \end{array} \Rightarrow \begin{array}{l} -29 \cdot \frac{4}{3} + 8x_2 = 16 \Rightarrow x_2 = \frac{41}{6} \\ 12 \cdot \left(-\frac{4}{3}\right) + 2 \cdot \frac{41}{6} + 4x_3 = 6 \Rightarrow x_3 = \frac{25}{12} \end{array}$$

0. Fall $\mathbb{L} = \left\{ \left(-\frac{4}{3} \mid \frac{41}{6} \mid \frac{25}{12} \right) \right\}$