Nr. 7) symmetrisch zur y-Achse $\Rightarrow f(x) = ax^4 + bx^2 + c$

$f'(x) = 4ax^3 + 2bx$

Schnitt y-Achse bei $y = -1 \Rightarrow f(0) = a \cdot 0 + b \cdot 0 + \underline{\underline{c = -1}}$

$T(1|-3) \Rightarrow f(1) = a + b - 1 = -3$

$\Rightarrow f'(1) = 4a + 2b = 0$

$$\begin{array}{rl|l} a + b & = -2 & \cdot 4 \\ 4a + 2b & = 0 & \cdot(-1) \end{array}$$

$$\begin{array}{rl} a + b & = -2 \Rightarrow \underline{\underline{a = -2 + 4 = 2}} \\ 2b & = -8 \Rightarrow \underline{\underline{b = -4}} \end{array}$$

$\underline{\underline{f(x) = 2x^4 - 4x^2 - 1}} \Rightarrow f(1) = -3$

$$\left.\begin{array}{l} f'(x) = 8x^3 - 8x \Rightarrow f'(1) = 0 \\ f''(x) = 24x^2 - 8 \Rightarrow f''(1) = +16 \end{array}\right\} \Rightarrow T(1|-3) \checkmark$$

Nr. 8) $f(x) = ax^3 + bx^2 + cx + d$; $f'(x) = 3ax^2 + 2bx + c$

a) $A(1|4) \Rightarrow f(1) = a + b + c + d = 4$

$B(-1|6) \Rightarrow f(-1) = -a + b - c + d = 6$

$C(-2|4) \Rightarrow f(-2) = -8a + 4b - 2c + d = 4$

Hochpunkt y-Achse $\Rightarrow f'(0) = 3a \cdot 0 + 2 \cdot b \cdot 0 + \underline{\underline{c = 0}}$

$$\begin{array}{rl|l|l} a + b + d & = 4 & \cdot 1 & \cdot 1 \\ -a + b + d & = 6 & \cdot(-1) & \\ -8a + 4b + d & = 4 & & \cdot(-1) \end{array}$$

$$\begin{array}{rl} a + b + d & = 4 \Rightarrow -1 - 3 + d = 4 \Rightarrow \underline{\underline{d = 4 + 1 + 3 = 8}} \\ 2a & = -2 \Rightarrow \underline{\underline{a = -1}} \\ 9a - 3b & = 0 \Rightarrow -3b = -9a = +9 \Rightarrow \underline{\underline{b = -3}} \end{array}$$

$\underline{\underline{f(x) = -1x^3 - 3x^2 + 8}}$

$$\left.\begin{array}{l} f'(x) = -3x^2 - 6x \Rightarrow f'(0) = 0 \\ f''(x) = -6x - 6 \Rightarrow f''(0) = -6 < 0 \end{array}\right\} \Rightarrow H(0|8) \checkmark$$

Nr. 8) $f(x) = ax^3 + bx^2 + cx + d$; $f'(x) = 3ax^2 + 2bx + c$

b) $A(2|2) \Rightarrow f(2) = 8a + 4b + 2c + d = 2 \quad | \cdot 1$

$B(3|9) \Rightarrow f(3) = 27a + 9b + 3c + d = 9 \quad | \cdot 1$

$T(1|1) \Rightarrow f(1) = a + b + c + d = 1 \quad | \cdot(-1) \quad | \cdot(-1)$

$f'(1) = 3a + 2b + c = 0$

$$\begin{aligned} a + b + c + d &= 1 \\ 26a + 8b + 2c &= 8 \quad | \cdot(-1) \\ 7a + 3b + c &= 1 \quad\quad\quad | \cdot(-1) \\ 3a + 2b + c &= 0 \quad | \cdot 2 \quad | \cdot 1 \end{aligned}$$

$$\begin{aligned} a + b + c + d &= 1 \\ 3a + 2b + c &= 0 \\ -20a - 4b &= -8 \quad | \cdot(-1) \\ -4a - b &= -1 \quad | \cdot 4 \end{aligned}$$

$$\begin{aligned} a + b + c + d &= 1 \Rightarrow \underline{\underline{d = -1 - 1 \cdot (-3) - 3 + 1 = 0}} \\ 3a + 2b + c &= 0 \Rightarrow \underline{\underline{c = 0 - 3 - 2 \cdot (-3) = 3}} \\ -4a - b &= -1 \Rightarrow -b = -1 + 4 \Rightarrow \underline{\underline{b = -3}} \\ 4a &= +4 \Rightarrow \underline{\underline{a = 1}} \end{aligned}$$

$f(x) = 1 \cdot x^3 - 3 \cdot x^2 + 3x$

$f'(x) = 3x^2 - 6 \cdot x + 3 \Rightarrow f'(1) = 3 \cdot 1 - 6 + 3 = 0$

$f''(x) = 6x - 6 \Rightarrow f''(1) = 6 \cdot 1 - 6 = 0$?

$\left.\begin{aligned} f'(x) &> 0 \text{ für } x < 1 \\ f'(x) &> 0 \text{ für } 1 < x \end{aligned}\right\} \Rightarrow$ kein VZW von $f'(x)$ an der Stelle 1 $\Rightarrow$ An der Stelle 1 ist kein Tiefpunkt sondern ein Sattelpunkt. $\Rightarrow$ Keine Lösung

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