Nr. 3.) a) $f(x) = 2\cdot\sin(x) + 1 \quad ; \quad x \in [0; 2\pi]$

$f'(x) = 2\cdot\cos(x) \quad ; \quad f''(x) = -2\sin(x) \quad ; \quad f'''(x) = -2\cos(x)$

Extrema notw. Bed. $f'(x) = 0 = 2\cos(x) \Rightarrow x_1 = \frac{\pi}{2} \vee x_2 = \frac{3}{2}\pi$

hinr. Bed. $f''(\frac{\pi}{2}) = -2\cdot\underbrace{\sin(\frac{\pi}{2})}_{=+1} < 0 \Rightarrow \underline{H}(\frac{\pi}{2} | 2\cdot 1 + 1) = \underline{(\frac{\pi}{2} | 3)}$

$f''(\frac{3}{2}\pi) = -2\cdot\underbrace{\sin(\frac{3}{2}\pi)}_{=-1} > 0 \Rightarrow \underline{T}(\frac{3}{2}\pi | 2\cdot(-1)+1) = \underline{(\frac{3}{2}\pi | -1)}$

Wendepunkte notw. Bed. $f''(x) = 0 = -2\cdot\sin(x)$

$\Rightarrow (x_3 = 0) \vee \underline{x_4 = \pi} \vee (x_5 = 2\pi)$

hinr. Bed $(f'''(0) = -2 \neq 0 \Rightarrow \cancel{W_1(0|1)}) \rightarrow$ Rand keine Änderung der Krümmung

$f'''(\pi) = -2\cdot(-1) = 2 \neq 0 \Rightarrow \underline{W_2(\pi | 1)}$

$(f'''(2\pi) = -2\cdot 1 = -2 \neq \Rightarrow \cancel{W_3(2\pi|1)})$

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b) $f(x) = -\cos(2x) - 1 \quad ; \quad x \in [0; \pi]$

$f'(x) = \sin(2x)\cdot 2 = 2\cdot\sin(2x)$

$f''(x) = 2\cdot\cos(2x)\cdot 2 = 4\cdot\cos(2x)$

$f'''(x) = 4\cdot(-\sin(2x))\cdot 2 = -8\cdot\sin(2x)$

Extrema: notw. Bed. $f'(x) = 2\cdot\sin(2x) = 0$

$\Rightarrow 2x = 0 \vee 2x = \pi \vee 2x = 2\pi$

$x_1 = 0 \vee x_2 = \frac{\pi}{2} \vee x_3 = \pi$

hinr. Bed. $f''(\frac{\pi}{2}) = 4\cdot\cos(2\cdot\frac{\pi}{2}) = 4\cdot(-1) = -4 < 0 \Rightarrow$

$\underline{H}(\frac{\pi}{2} | -\cos(2\cdot\frac{\pi}{2}) - 1) = (\frac{\pi}{2} | 1 - 1) = \underline{(\frac{\pi}{2} | 0)}$

$\underline{T_1(0|-2)} \vee \underline{T_2(\pi|-2)}$ liegen am Rand des Intervalls $\Rightarrow$ Randextrema

Wendestellen: $f''(x) \overset{!}{=} 0 = 4\cos(2x)$

$\Rightarrow 2x = \frac{\pi}{2} \vee 2x = \frac{3}{2}\pi$

$x_4 = \frac{\pi}{4} \vee x_5 = \frac{3}{4}\pi$

$f'''(\frac{\pi}{4}) = -8\cdot 1 \neq 0 \quad ; \quad f'''(\frac{3}{4}\pi) = -8\cdot(-1) \neq 0 \Rightarrow$ Wendestellen $\underline{\frac{\pi}{4} ; \frac{3}{4}\pi}$

O. Fell

Nr. 4) a) $f(x) = x$, $f(x) = 2^x$

b) $f(x) = \sin(x)$

c) $f(x) = x^2$

d) $f(x) = x^3 - 2x$