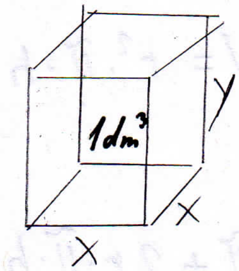
Nr. 5)

a)



$x > 0$

Nebenbedingung : $V = x^2 \cdot y = 1$

$\Rightarrow y = \frac{1}{x^2}$

Zielfunktion : $O = 2x^2 + 4 \cdot xy$

$O(x) = 2x^2 + 4 \cdot x \cdot \left(\frac{1}{x^2}\right) = 2x^2 + 4 \cdot \frac{1}{x}$

Extrema : $O'(x) = 4x - \frac{4}{x^2} = 0 \Rightarrow 4x = \frac{4}{x^2} \mid \cdot x^2$

$4x^3 = 4 \mid :4 \Rightarrow x^3 = 1 \Rightarrow x_E = 1$

hinr. Bed $O''(x) = 4 + \frac{8}{x^3} \Rightarrow O''(1) = 4 + \frac{8}{1^3} > 0$

$\Rightarrow$ Minimum der Oberfläche für $x = 1\,dm$ und $y = 1\,dm$.

b) Oben offen : Nebenbedingung $V = x^2 \cdot y = 1 \Rightarrow y = \frac{1}{x^2}$

Zielfunktion : $A = x^2 + 4 \cdot xy$

$A(x) = x^2 + 4 \cdot x \cdot \frac{1}{x^2} = x^2 + 4 \cdot \frac{1}{x}$

Extrema: $A'(x) = 2x - \frac{4}{x^2} = 0$ notw. Bed.

$\Rightarrow 2x = \frac{4}{x^2} \mid \cdot x^2 \Rightarrow 2x^3 = 4 \mid :2 \Rightarrow x^3 = 2 \mid \sqrt[3]{\ }$

$\Rightarrow x_E = \sqrt[3]{2} \approx 1{,}26$

hinr. Bed $A''(x) = 2 + \frac{8}{x^3} = 2 + \frac{8}{(\sqrt[3]{2})^3} = 2 + \frac{8}{2} = 6 > 0$

$\Rightarrow$ Minimum der Fläche für $x_E = \sqrt[3]{2} \approx 1{,}26\,(dm)$

und $y = \frac{1}{\sqrt[3]{2^2}} = \frac{1}{\sqrt[3]{4}} \approx 0{,}63\,(dm)$

O. Fell