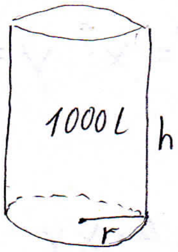
Nr. 6)



$r > 0$

Nebenbedingung: $V = r^2 \cdot \pi \cdot h = 1000$

$\Rightarrow h = \dfrac{1000}{r^2 \cdot \pi}$

Zielfunktion: $A = r^2 \cdot \pi + 2r \cdot \pi \cdot h$

$\underline{\underline{A(r)}} = r^2 \cdot \pi + 2r \cdot \pi \cdot \dfrac{1000}{r^2 \cdot \pi} = \underline{\underline{r^2 \cdot \pi + \dfrac{2000}{r}}}$

Extrema notw. Bed $A'(r) = 2 \cdot \pi \cdot r - \dfrac{2000}{r^2} = 0$

$\Rightarrow 2\pi \cdot r = \dfrac{2000}{r^2} \;\Big|\cdot r^2 \Rightarrow 2\pi r^3 = 2000 \;\Big|: 2\pi$

$\Rightarrow r^3 = \dfrac{1000}{\pi} \;\Big|\sqrt[3]{} \Rightarrow \underline{\underline{r_E = \sqrt[3]{\dfrac{1000}{\pi}} = \dfrac{10}{\sqrt[3]{\pi}}}}$

hinr. Bed $A''(r) = 2\pi + \dfrac{4000}{r^3} \Rightarrow A''\left(\dfrac{10}{\sqrt[3]{\pi}}\right) = 2\pi + \dfrac{4000}{\frac{1000}{\pi}} > 0$

Die <u>Fläche wird minimal</u> wenn $\underline{r_E} = \dfrac{10}{\sqrt[3]{\pi}} \approx 6{,}83$ (dm)

und $h = \dfrac{1000}{\left(\frac{10}{\sqrt[3]{\pi}}\right)^2 \cdot \pi} = \dfrac{1000 \cdot \pi^{\frac{2}{3}}}{100 \cdot \pi} = 10 \cdot \pi^{\frac{2}{3}} \cdot \pi^{-1} = 10 \cdot \pi^{-\frac{1}{3}} = \underline{\underline{\dfrac{10}{\sqrt[3]{\pi}}}} \approx 6{,}83$ (dm)

ist. Radius und Höhe sind gleich.

O. Fell