

Nr. 15) a) $f(x) = 3^x = 7 \quad | \ln$

$$\ln(3^x) = \ln(7)$$

$$x \cdot \ln(3) = \ln(7) \quad | : \ln(3)$$

$$x = \frac{\ln(7)}{\ln(3)} \approx \underline{\underline{1,77}}$$

$$f(x) = 3^x = e^{\ln(3^x)} = e^{x \cdot \ln(3)}$$

$$f'(x) = e^{x \cdot \ln(3)} \cdot \ln(3) = \ln(3) \cdot 3^x$$

b) $f(x) = 4 + 2^{-x} = 7 \quad | -4$

$$2^{-x} = 3 \quad | \ln \Rightarrow -x \cdot \ln(2) = \ln(3) \quad | : (-\ln(2))$$

$$x = \frac{-\ln(3)}{\ln(2)} \approx \underline{\underline{-1,58}}$$

$$f(x) = 4 + e^{\ln(2^{-x})} = 4 + e^{-x \cdot \ln(2)}$$

$$f'(x) = \underbrace{e^{-x \cdot \ln(2)}}_{= 2^{-x}} \cdot (-\ln(2)) = \underline{\underline{-\ln(2) \cdot 2^{-x}}}$$

c) $f(x) = 8 \cdot 5^x - 25^x = 7$

$$8 \cdot 5^x - (5^2)^x = 7 \Rightarrow 8 \cdot 5^x - 5^{2x} = 7 \quad | \text{Sub: } 5^x = u$$

$$8u - u^2 = 7 \quad | +u^2 - 8u$$

$$u^2 - 8u + 7 = 0 \Rightarrow u_{1,2} = 4 \pm \sqrt{16 - 7} = 4 \pm 3$$

$$u_1 = 7 \quad | \text{Rück. Sub}$$

$$5^x = 7 \quad | \ln$$

$$x \ln(5) = \ln(7) \quad | : \ln(5)$$

$$x_1 = \frac{\ln(7)}{\ln(5)} \approx \underline{\underline{1,21}}$$

$$u_2 = 1 \quad | \text{Rück. Sub}$$

$$5^x = 1 \quad | \ln$$

$$x \cdot \ln(5) = \ln(1) = 0 \quad | : \ln(5)$$

$$x_2 = \frac{0}{\ln(5)} = 0$$

$$\text{Nr. 15) c) } f(x) = 8 \cdot 5^x - 25^x = 8 \cdot 5^x - (5^2)^x$$

$$f(x) = 8 \cdot 5^x - 5^{2x} = 8 \cdot e^{\ln(5^x)} - e^{\ln(5^{2x})}$$

$$f(x) = 8 \cdot e^{x \cdot \ln(5)} - e^{2x \cdot \ln(5)}$$

Ableitung:

$$f'(x) = 8 \cdot e^{x \cdot \ln(5)} \cdot \ln(5) - e^{2x \cdot \ln(5)} \cdot (2 \cdot \ln(5))$$

$$f'(x) = 2 \cdot \ln(5) \cdot \underbrace{e^{x \cdot \ln(5)}}_{=5^x} \cdot \left(4 - \underbrace{e^{2x \cdot \ln(5)}}_{=5^{2x}} \right)$$

$$\underline{\underline{f'(x) = 2 \cdot \ln(5) \cdot 5^x \cdot (4 - 5^{2x})}}$$

$$\text{d) } f(x) = 49^{x^2 - 0,5x} = 7 \mid \ln$$

$$(x^2 - 0,5x) \cdot \ln(49) = \ln(7) \mid : \ln(49)$$

$$x^2 - 0,5x = \frac{\ln(7)}{\ln(49)} = \frac{\ln(7)}{\ln(7^2)} = \frac{\ln(7)}{2 \cdot \ln(7)} = \frac{1}{2}$$

$$x^2 - 0,5x - \frac{1}{2} = 0 \Rightarrow x_{1,2} = \frac{1}{4} \pm \sqrt{\frac{1}{16} + \frac{1}{2}} = \frac{1}{4} \pm \sqrt{\frac{9}{16}}$$

$$\underline{\underline{x_1 = \frac{1}{4} + \frac{3}{4} = 1}} \quad \vee \quad \underline{\underline{x_2 = \frac{1}{4} - \frac{3}{4} = -\frac{1}{2}}}$$

$$f(x) = e^{\ln(49^{x^2 - 0,5x})} = e^{(x^2 - 0,5x) \cdot \ln(49)}$$

$$f'(x) = \underbrace{e^{(x^2 - 0,5x) \cdot \ln(49)}}_{f(x) = 49^{x^2 - 0,5x}} \cdot (2x - 0,5) \cdot \ln(49)$$

$$f'(x) = 49^{x^2 - 0,5x} \cdot (2x - 0,5) \cdot \ln(49)$$

$$\underline{\underline{f'(x) = (2x - 0,5) \cdot \ln(49) \cdot 49^{x^2 - 0,5x}}}$$