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Nr. 11) a) $f(x) = x - \ln(x)$, Extrema $f'(x) = 0$ notw. Bed

$f'(x) = 1 - \frac{1}{x} = 0 \mid \cdot x \Rightarrow x - 1 = 0 \Rightarrow \underline{x_1 = 1}$

hinr. Bed. $f''(x) = +\frac{1}{x^2}$; $f''(1) = 1 > 0 \Rightarrow \underline{T(1|1)}$

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b) $f(x) = x \cdot \ln(x) - 2x$; $f'(x) = 1 \cdot \ln(x) + x \cdot \frac{1}{x} - 2$

$f'(x) = \ln(x) - 1 = 0 \Rightarrow \ln(x) = 1 \mid e^{\uparrow} \Rightarrow \underline{x_1 = e^1 = e}$

hinr. Bed. $f''(x) = \frac{1}{x}$; $f''(e) = \frac{1}{e} > 0 \Rightarrow T(e \mid e \cdot 1 - 2e)$

$\underline{T(e \mid -e)}$

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c) $f(x) = x^2 \cdot \ln(x)$; $\mathbb{D}_f = \mathbb{R}^+ \setminus \{0\}$

$f'(x) = 2x \cdot \ln(x) + x^2 \cdot \frac{1}{x} = 2x \cdot \ln(x) + x$

$\underline{f'(x) = x \cdot (2 \cdot \ln(x) + 1) = 0}$; $x_1 = 0 \notin \mathbb{D}_f \Rightarrow$ keine Lösung

$2 \cdot \ln(x) + 1 = 0 \mid -1 \Rightarrow 2\ln(x) = -1 \mid :2$

$\ln(x) = -\frac{1}{2} \mid e^{\uparrow} \Rightarrow \underline{x_2 = e^{-\frac{1}{2}} = \frac{1}{e^{\frac{1}{2}}} = \frac{1}{\sqrt{e}}}$

hinr. Bed: $f''(x) = 2 \cdot \ln(x) + 2x \cdot \frac{1}{x} + 1$

$f''(x) = 2 \cdot \ln(x) + 3$

$f''\left(\frac{1}{\sqrt{e}}\right) = 2 \cdot \ln\left(e^{-\frac{1}{2}}\right) + 3 = 2 \cdot \left(-\frac{1}{2}\right) + 3 = 2 > 0$

$\Rightarrow \underline{T\left(\frac{1}{\sqrt{e}} \mid \left(\frac{1}{\sqrt{e}}\right)^2 \cdot \ln\left(\frac{1}{\sqrt{e}}\right)\right) = \left(\frac{1}{\sqrt{e}} \mid \frac{1}{e} \cdot \left(-\frac{1}{2}\right)\right) = \left(\frac{1}{\sqrt{e}} \mid -\frac{1}{2e}\right)}$