

Nr. 12) a)  $f(x) = (x+2)^2 \Rightarrow F(x) = \frac{(x+2)^3}{3} + C$

$$F(0) = 1 = \frac{(0+2)^3}{3} + C \Rightarrow 1 - \frac{8}{3} = C \Leftrightarrow C = -\frac{5}{3}$$

$$F(x) = \frac{(x+2)^3}{3} - \frac{5}{3}$$


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b)  $f(x) = \frac{1}{x+1} \Rightarrow F(x) = \ln(|x+1|) + C$

$$F(0) = 1 = \underbrace{\ln(|0+1|)}_{=0} + C \Rightarrow C = 1$$

$$F(x) = \ln(|x+1|) + 1$$


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c)  $f(t) = 2 \cdot e^{0,5 \cdot t} \Rightarrow F(t) = 2 \cdot e^{0,5 \cdot t} \cdot \frac{1}{\frac{1}{2}} + C$

$$F(t) = 4 \cdot e^{0,5t} + C \Rightarrow F(0) = 4 \cdot e^0 + C = 1$$

$$\Rightarrow C = 1 - 4 = -3 \Rightarrow F(t) = 4 \cdot e^{0,5 \cdot t} - 3$$


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d)  $f(t) = \cos(5t) \Rightarrow F(t) = \sin(5t) \cdot \frac{1}{5} + C$

$$F(0) = \underbrace{\sin(5 \cdot 0)}_{=0} \cdot \frac{1}{5} + C = 1 \Rightarrow C = 1$$

$$F(t) = \frac{1}{5} \sin(5t) + 1$$


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Nr. 15)  $f(x) = \frac{x^2 + 2x}{x^4} = \frac{1}{x^2} + \frac{2}{x^3} = x^{-2} + 2 \cdot x^{-3}$

a)

$$F(x) = \frac{x^{-1}}{-1} + 2 \cdot \frac{x^{-2}}{-2} = \frac{-1}{x} - \frac{1}{x^2}$$

Nr. 15 b)  $f(x) = \frac{x^3 + 1}{2x^2} = \frac{1}{2}x + \frac{1}{2}x^{-2}$

$$F(x) = \frac{1}{2} \cdot \frac{x^2}{2} + \frac{1}{2} \cdot \frac{x^{-1}}{-1} = \frac{1}{4}x^2 - \frac{1}{2x}$$


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c)  $f(x) = \frac{1+x+x^3}{3x^3} = \frac{1}{3}x^{-3} + \frac{1}{3}x^{-2} + \frac{1}{3}$

$$F(x) = \frac{1}{3} \cdot \frac{x^{-2}}{-2} + \frac{1}{3} \cdot \frac{x^{-1}}{-1} + \frac{1}{3}x = -\frac{1}{6x^2} - \frac{1}{3x} + \frac{1}{3}x$$


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d)  $f(x) = \frac{(2x+1)^2 - 1}{x} = \frac{4x^2 + 4x + 1 - 1}{x} = 4x + 4$

$$F(x) = 4 \cdot \frac{x^2}{2} + 4x = \underline{\underline{2x^2 + 4x}}$$


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Nr. 16)

$$\int_a^b f(x) dx + \int_b^c f(x) dx = \left[ F(x) \right]_a^b + \left[ F(x) \right]_b^c$$


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$$= \cancel{F(b)} - F(a) + F(c) - \cancel{F(b)} = F(c) - F(a)$$

$$= \int_a^c f(x) dx$$


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