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Nr. 1) Fläche I $\quad A_I = A_2 + A_3 = \int\limits_{-1}^{1} f(x)\,dx$

Fläche $\underline{II}$ $\quad A_{\underline{II}} = A_4 + A_2 + A_3 + A_5 = \int\limits_{-2}^{2} \big(f(x) - g(x)\big)\,dx$

Fläche $\underline{IV}$ $\quad A_{\underline{IV}} = A_3 = \int\limits_{0}^{1} f(x)\,dx$

Fläche $\underline{V}$ $\quad A_{\underline{V}} = A_1 = -\int\limits_{-2}^{-1} f(x)\,dx$

Nr. 2.)

a) $A = \int\limits_{-2}^{-1} (x^2-1)\,dx - \int\limits_{-1}^{0} (x^2-1)\,dx = \left[\frac{x^3}{3} - x\right]_{-2}^{-1} + \left[\frac{x^3}{3} - x\right]_{0}^{-1}$

$A = -\frac{1}{3} + 1 - \left\{\frac{-8}{3} + 2\right\} + \left(\frac{-1}{3} + 1\right) - \left\{\frac{0}{3} - 0\right\} = \underline{\underline{2}}$ (FE ≙ Flächeneinheiten)

b) Schnitt Schaubild mit x-Achse

$\frac{1}{x} - 2 = 0 \Rightarrow \frac{1}{x} = 2 \;\Big|\cdot \frac{x}{2} \Rightarrow x_1 = \frac{1}{2}$

$A = -\int\limits_{\frac{1}{2}}^{2} \left(\frac{1}{x} - 2\right)dx = -\Big[\ln(|x|) - 2x\Big]_{\frac{1}{2}}^{2} = -\left(\ln(2) - 4 - \left\{\ln\left(\tfrac{1}{2}\right) - 1\right\}\right) =$

$-\ln(2) + 4 + \ln\left(\frac{1}{2}\right) - 1 = -\ln(2) + 4 + \big(\underbrace{\ln(1)}_{=0} - \ln(2)\big) - 1 = \underline{\underline{-2\ln(2) + 3}}$

c) Schnitt f mit g

$-x^2 + 1 = -3 \Rightarrow x^2 = 4 \Rightarrow x_{1,2} = \overset{+}{(-)}\sqrt{4} = +2$

$A = \int\limits_{0}^{2} \big(f(x) - g(x)\big)\,dx = \int\limits_{0}^{2} \big(-x^2 + 1 - (-3)\big)\,dx = \int\limits_{0}^{2} (-x^2 + 4)\,dx = \left[-\frac{x^3}{3} + 4x\right]_{0}^{2} =$

$-\frac{8}{3} + 8 - \left\{-\frac{0^3}{3} + 4\cdot 0\right\} = \underline{\underline{\frac{16}{3}}}$

Nr. 2) d) $A = -\int_0^1 (e^{x-1} - 1)dx + \int_1^2 (e^{x-1} - 1)dx =$

$-\left[e^{x-1} - 1x\right]_0^1 + \left[e^{x-1} - 1x\right]_1^2 = -\left[(e^0 - 1\cdot 1) - \{e^{0-1} - 0\}\right]$

$+ \left(e^{2-1} - 2 - \{e^0 - 1\}\right) = 0 + \frac{1}{e} + e - 2 - 1 + 1 = \underline{\underline{\frac{1}{e} + e - 2}}$

Nr. 3) a) $f(x) = 3x^2 - 3$ Integrationsgrenzen $\hat{=}$ Nullstellen

$f(x) = 3x^2 - 3 = 0 \Rightarrow x^2 = 1 \Rightarrow x_{1,2} = \pm 1$

$A = -\int_{-1}^{+1} (3x^2 - 3)\,dx = -\left[3 \cdot \frac{x^3}{3} - 3x\right]_{-1}^{+1} = -\left(1^3 - 3\cdot 1 - \{-1 - 3\cdot(-1)\}\right) = \underline{\underline{4}}$

b) $f(x) = 4 - x^2 = 0 \Rightarrow x^2 = 4 \Rightarrow x_{1,2} = \pm 2$

$A = \int_{-2}^{2} (4 - x^2)\,dx = \left[4x - \frac{x^3}{3}\right]_{-2}^{2} = 4\cdot 2 - \frac{8}{3} - \left\{4\cdot(-2) - \frac{(-2)^3}{3}\right\}$

$= 8 - \frac{8}{3} + 8 - \frac{8}{3} = 16 - \frac{16}{3} = \underline{\underline{\frac{32}{3}}}$

c) $f(x) = 5x^4 - 80 = 0 \Rightarrow x^4 = \frac{80}{5} = 16 \Rightarrow x_{1,2} = \pm 2$

$A = \left|\int_{-2}^{+2} (5x^4 - 80)\,dx\right| = -\left[5 \cdot \frac{x^5}{5} - 80x\right]_{-2}^{+2} = -\left(2^5 - 80\cdot 2 - \{(-2)^5 - 80\cdot(-2)\}\right)$

$= -(-256) = \underline{\underline{+256}}$

d) $f(x) = 0{,}5x^2 - 3x = x\cdot(0{,}5x - 3) = 0 \Rightarrow x_1 = 0 \vee x_2 = 6$

$A = \left|\int_0^6 (0{,}5x^2 - 3x)\,dx\right| = -\left(\left[\frac{1}{2}\cdot\frac{x^3}{3} - 3\frac{x^2}{2}\right]_0^6\right) =$

$-(36 - 54 - \{0\}) = -(-18) = \underline{\underline{+18}}$

Nr. 3) e) $f(x) = x^2 + x = x(x+1) = 0 \Rightarrow x_1 = 0 \lor x_2 = -1$

$$A = \left| \int_{-1}^{0} (x^2 + x)\,dx \right| = -\left[\frac{x^3}{3} + \frac{x^2}{2} \right]_{-1}^{0} = -\left(\frac{0}{3} + \frac{0}{2} - \left\{ \frac{-1}{3} + \frac{1}{2} \right\} \right)$$

$$-\left(+ \frac{1}{3} - \frac{1}{2} \right) = -\left(- \frac{1}{6} \right) = +\frac{1}{6}$$

f) $f(x) = x^4 - 4x^2 = x^2 \cdot (x^2 - 4) = 0 \Rightarrow x_1 = 0 \lor x_2 = -2 \lor x_3 = +2$

$$A = \left| \int_{-2}^{0} (x^4 - 4x^2)\,dx \right| + \left| \int_{0}^{-2} (x^4 - 4x^2)\,dx \right| =$$

$$-\left(\left[\frac{x^5}{5} - 4 \cdot \frac{x^3}{3} \right]_{-2}^{0} \right) - \left(\left[\frac{x^5}{5} - 4 \cdot \frac{x^3}{3} \right]_{0}^{2} \right) =$$

$$-\left(0 - 0 - \left\{ -\frac{32}{5} - 4\frac{(-8)}{3} \right\} \right) - \left(\frac{32}{5} - 4 \cdot \frac{8}{3} - \{0\} \right) = \frac{64}{15} + \frac{64}{15} = \frac{128}{15}$$

g) $f(x) = (x-1) \cdot (x+2) = 0 \Rightarrow x_1 - 2 \lor x_2 = 1$

$f(x) = x^2 + x - 2$

$$A = \left| \int_{-2}^{1} (x^2 + x - 2)\,dx \right| = -\left[\frac{x^3}{3} + \frac{x^2}{2} - 2x \right]_{-2}^{1} =$$

$$-\left(\frac{1}{3} + \frac{1}{2} - 2 - \left\{ -\frac{8}{3} + \frac{4}{2} + 4 \right\} \right) = -\left(-\frac{7}{6} - \left\{ \frac{10}{3} \right\} \right) = \frac{9}{2}$$

h) $f(x) = 3(x+2)(x-3) = 0 \Rightarrow x_1 = -2 \lor x_2 = 3$

$f(x) = 3(x^2 - x - 6)$

$$A = \left| \int_{-2}^{3} 3(x^2 - x - 6)\,dx \right| = -3\left[\frac{x^3}{3} - \frac{x^2}{2} - 6x \right]_{-2}^{3} =$$

$$-3 \cdot \left(9 - \frac{9}{2} - 18 - \left\{ -\frac{8}{3} - \frac{4}{2} + 12 \right\} \right) = \frac{125}{2}$$

i) $f(x) = (x-1)^2 - 1 = 0 \Rightarrow (x-1)^2 = 1 \Rightarrow x - 1 = \pm 1 \Rightarrow x_1 = 0 \lor x_2 = 2$

$$A = \left| \int_{0}^{2} ((x^2 - 1) - 1)\,dx \right| = \left| \int_{0}^{2} (x^2 - 2x + 1 - 1)\,dx \right| = \left| \int_{0}^{2} (x^2 - 2x)\,dx \right| =$$

$$-\left[\frac{x^3}{3} - 2 \cdot \frac{x^2}{2} \right]_{0}^{2} = -\left(\frac{8}{3} - 4 - \{0\} \right) = -\left(-\frac{4}{3} \right) = \frac{4}{3}$$

O. Fell