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Nr. 4) a) $f(x) = x^2$; $g(x) = 9$

Integrationsgrenzen $\hat{=}$ Schnittstellen der Funktionen.

$f(x) = g(x) \Leftrightarrow x^2 = 9 \Rightarrow x_1 = -3 \vee x_2 = 3$

$$A = \int_{-3}^{3} (g(x) - f(x))\,dx = \int_{-3}^{3} (9 - x^2)\,dx = \left[9x - \frac{x^3}{3}\right]_{-3}^{3} =$$

$$9 \cdot 3 - \frac{27}{3} - \left\{9 \cdot (-3) - \frac{(-3)^3}{3}\right\} = 27 - 9 + 27 - 9 = 54 - 18 = \underline{\underline{36}}$$

b) $f(x) = x^2$ $g(x) = x$

$f(x) = g(x) \Rightarrow x^2 = x \Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0 \Rightarrow x_1 = 0 \vee x_2 = 1$

$$A = \int_{0}^{1} (x - x^2)\,dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{0}^{1} = \frac{1}{2} - \frac{1}{3} - \{0\} = \underline{\underline{\frac{1}{6}}}$$

c) $f(x) = x^2$; $g(x) = -2x$

$f(x) = g(x) \Rightarrow x^2 = -2x \Rightarrow x^2 + 2x = 0 \Rightarrow x(x+2) = 0 \Rightarrow x_1 = -2 \vee x_2 = 0$

$$A = \int_{-2}^{0} (-2x - x^2)\,dx = \left[-2 \cdot \frac{x^2}{2} - \frac{x^3}{3}\right]_{-2}^{0} = 0 - \left\{-(-2)^2 - \frac{(-8)}{3}\right\} = \underline{\underline{\frac{4}{3}}}$$

d) $f(x) = x^2$ $g(x) = 2x + 3$

$f(x) = g(x) \Rightarrow x^2 = 2x + 3 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow x_{1,2} = \frac{+2 \pm \sqrt{4 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1}$

$x_{1,2} = \frac{+2 \pm \sqrt{16}}{2} \Rightarrow x_1 = 3 \vee x_2 = -1$

$$A = \int_{-1}^{3} (2x + 3 - x^2)\,dx = \left[\cancel{2} \cdot \frac{x^2}{\cancel{2}} + 3x - \frac{x^3}{3}\right]_{-1}^{3} = 9 + 9 - 9 - \left\{1 - 3 + \frac{1}{3}\right\} = \underline{\underline{\frac{32}{3}}}$$

e) $f(x) = x^2$ $g(x) = 2 - x^2$

$f(x) = g(x) \Rightarrow x^2 = 2 - x^2 \Rightarrow 2x^2 - 2 = 0 \Rightarrow x_{1,2} = \pm 1$

$$A = \int_{-1}^{1} (g(x) - f(x))\,dx = \int_{-1}^{1} (2 - x^2 - x^2)\,dx = \int_{-1}^{1} (2 - 2x^2)\,dx =$$

$$\left[2x - 2 \cdot \frac{x^3}{3}\right]_{-1}^{1} = 2 - \frac{2}{3} - \left\{-2 + \frac{2}{3}\right\} = 2 - \frac{2}{3} + 2 - \frac{2}{3} = 4 - \frac{4}{3} = \underline{\underline{\frac{8}{3}}}$$

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Nr. 4 f) $f(x)=x^2 \quad g(x)=\frac{1}{2}x^2+\frac{9}{2}$

$f(x)=g(x) \Rightarrow x^2=\frac{1}{2}x^2+\frac{9}{2} \Rightarrow \frac{1}{2}x^2=\frac{9}{2} \mid \cdot 2 \Rightarrow x_{1,2}=\pm 3$

$A=\int_{-3}^{3}\left(\frac{1}{2}x^2+\frac{9}{2}-x^2\right)dx=\int_{-3}^{3}\left(-\frac{1}{2}x^2+\frac{9}{2}\right)dx=$

$\left[-\frac{1}{2}\cdot\frac{x^3}{3}+\frac{9}{2}x\right]_{-3}^{3}=-\frac{9}{2}+\frac{27}{2}-\left\{+\frac{9}{2}-\frac{27}{2}\right\}=\underline{\underline{18}}$

g) $f(x)=x^2 ; g(x)=x^4-x^2+1$

$f(x)=g(x) \Rightarrow x^2=x^4-x^2+1 \Rightarrow x^4-2x^2+1=0 \quad |\text{ Sub: } x^2=u$

$u^2-2u+1=0 \Rightarrow u_{1,2}=\frac{+2\pm\sqrt{4-4\cdot1\cdot1}}{1\cdot 2}=\pm 1$

Rück Sub: $x^2=1 \quad \vee \; x^2=-1$ ↯

$x_{1,2}=\pm 1$

$A=\int_{-1}^{1}(g(x)-f(x))dx=\int_{-1}^{1}(x^4-x^2+1-x^2)dx=\int_{-1}^{1}(x^4-2x^2+1)dx=$

$\left[\frac{x^5}{5}-2\cdot\frac{x^3}{3}+1\cdot x\right]_{-1}^{1}=\frac{1}{5}-\frac{2}{3}+1-\left\{-\frac{1}{5}+\frac{2}{3}-1\right\}=\underline{\underline{\frac{16}{15}}}$

h) $f(x)=x^2 \quad g(x)=x^3$

$f(x)=g(x) \Rightarrow x^2=x^3 \Rightarrow x^3-x^2=0 \Rightarrow x^2(x-1)=0 \Rightarrow x_1=0 \vee x_2=1$

$A=\int_{0}^{1}(x^2-x^3)dx=\left[\frac{x^3}{3}-\frac{x^4}{4}\right]_0^1=\frac{1}{3}-\frac{1}{4}-\{0\}=\underline{\underline{\frac{1}{12}}}$

O. Fell