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Nr. 7) a)
$$A=\int_{-1}^{1}(f(x)-g(x))\,dx=\int_{-1}^{1}(x^2+4-(x+3))\,dx=\int_{-1}^{1}(x^2-x+1)\,dx=$$
$$\left[\frac{x^3}{3}-\frac{x^2}{2}+x\right]_{-1}^{1}=\frac{1}{3}-\frac{1}{2}+1-\left\{-\frac{1}{3}-\frac{1}{2}-1\right\}=\underline{\underline{\frac{8}{3}}}$$

b) $f(x)=x^2+4 \quad g(x)=2x+4$; $f(x)=g(x) \Rightarrow x_1=0 \vee x_2=2$
$$A=\int_{-1}^{0}(f(x)-g(x))\,dx+\int_{0}^{1}(g(x)-f(x))\,dx=\int_{-1}^{0}(x^2+4-(2x+4))\,dx+\int_{0}^{1}2x+4-(x^2+4)\,dx$$
$$=\int_{-1}^{0}(x^2-2x)\,dx+\int_{0}^{1}(2x-x^2)\,dx=\left[\frac{x^3}{3}-\cancel{2}\cdot\frac{x^2}{\cancel{2}}\right]_{-1}^{0}+\left[\cancel{2}\cdot\frac{x^2}{\cancel{2}}-\frac{x^3}{3}\right]_{0}^{1}=$$
$$0-\left\{-\frac{1}{3}-1\right\}+1-\frac{1}{3}-\{0\}=\frac{4}{3}+\frac{2}{3}=\frac{6}{3}=\underline{\underline{2}}$$

c) $f(x)=\frac{1}{x} \quad g(x)=x$; $x=1$; $x=e$ Schnittstellen (-1); $+1$
$$A=\int_{1}^{e}(g(x)-f(x))\,dx=\int_{1}^{e}\left(x-\frac{1}{x}\right)dx=\left[\frac{x^2}{2}-\ln(|x|)\right]_{1}^{e}=$$
$$\frac{e^2}{2}-\ln(e)-\left\{\frac{1}{2}-\ln(1)\right\}=\frac{e^2}{2}-1-\frac{1}{2}+0=\underline{\underline{\frac{e^2}{2}-\frac{3}{2}}}$$

d) $f(x)=\sin(x)$; $g(x)=\cos(x)$; $x=\frac{\pi}{2}$; $x=\pi$ kein Schnitt in $[\frac{\pi}{2};\pi]$
$$A=\int_{\frac{\pi}{2}}^{\pi}(\sin(x)-\cos(x))\,dx=\left[-\cos(x)-\sin(x)\right]_{\frac{\pi}{2}}^{\pi}=$$
$$-\cos(\pi)-\sin(\pi)-\left\{-\cos\left(\frac{\pi}{2}\right)-\sin\left(\frac{\pi}{2}\right)\right\}=1-0-0+1=\underline{\underline{2}}$$

e) $f(x)=\frac{1}{x}$; $g(x)=\frac{1}{e}x$; $x=1$; $x=e$

Schnitt $\frac{1}{x}=\frac{1}{e}x \mid\cdot x \Rightarrow 1=\frac{x^2}{e}\Rightarrow x^2=e\Rightarrow x_1=+\sqrt{e}\ \vee (x_2=-\sqrt{e})$
$$A=\int_{1}^{\sqrt{e}}\left(\frac{1}{x}-\frac{1}{e}x\right)dx+\int_{\sqrt{e}}^{e}\left(\frac{1}{e}x-\frac{1}{x}\right)dx=\left[\ln(|x|)-\frac{1}{e}\cdot\frac{x^2}{2}\right]_{1}^{\sqrt{e}}+\left[\frac{1}{e}\frac{x^2}{2}-\ln(|x|)\right]_{\sqrt{e}}^{e}$$
$$=\frac{1}{2}-\frac{1}{2}-\left\{\ln(1)-\frac{1}{2e}\right\}+\frac{e}{2}-\ln(e)-\left\{\frac{1}{2}-\frac{1}{2}\right\}=\underline{\underline{+\frac{1}{2e}+\frac{e}{2}-1}}$$

Nr. 7 f) $f(x) = \sin(x)$; $g(x) = \cos(x) + 1$; $x = \pi$ $x = 2\pi$

Schnitt im Intervall $[\pi; 2\pi] \Rightarrow x = \pi$

$f(x) = \cos(x) \Rightarrow \sin(\pi) = \cos(\pi) + 1 \Leftrightarrow 0 = -1 + 1$ ✓

$$A = \int_{\pi}^{2\pi} (\cos(x) + 1 - \sin(x))\,dx = \Big[\sin(x) + x - (-\cos(x))\Big]_{\pi}^{2\pi} =$$

$$\sin(2\pi) + 2\pi + \cos(2\pi) - \{\sin(\pi) + \pi + \cos(\pi)\} =$$

$$0 + 2\pi + 1 - \{0 + \pi - 1\} = \underline{\underline{\pi + 2}}$$

Nr. 8 a) $f(x) = 2x^2 - 3x$ $g(x) = 3x - x^2$

Schnitt $f(x) = g(x) \Rightarrow 2x^2 - 3x = 3x - x^2 \Rightarrow 3x^2 - 6x = 0$

$\Rightarrow x(3x - 6) = 0 \Rightarrow x_1 = 0 \lor x_2 = 2$

$$A = \int_0^2 (g(x) - f(x))\,dx = \int_0^2 (3x - x^2 - (2x^2 - 3x))\,dx$$

$$= \int_0^2 (-3x^2 + 6x)\,dx = \Big[-\cancel{3} \cdot \frac{x^3}{\cancel{3}} + 6 \cdot \frac{x^2}{2}\Big]_0^2 = -8 + 12 - \{0\} = \underline{\underline{4}}$$

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b) Schnitt $f(x) = g(x) \Leftrightarrow x^2 - 4 = x + 2 \Rightarrow x^2 - x - 6 = 0$

$\Rightarrow x_{1,2} = \frac{1 \pm \sqrt{1 - 4 \cdot 1 \cdot (-6)}}{2 \cdot 1} = \frac{1 \pm \sqrt{25}}{2} \Rightarrow x_1 = -2 \lor x_2 = 3$

$$A = \int_{-2}^{3} (g(x) - f(x))\,dx = \int_{-2}^{3} (x + 2 - (x^2 - 4))\,dx = \int_{-2}^{3} (-x^2 + x + 6)\,dx =$$

$$\Big[-\frac{x^3}{3} + \frac{x^2}{2} + 6x\Big]_{-2}^{3} = -9 + \frac{9}{2} + 18 - \{+\frac{8}{3} + \frac{4}{2} - 12\} = \underline{\underline{\frac{125}{6}}}$$